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Reform of Teaching a Trigonometry Course

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REFORM OF TEACHING A TRIGONOMETRY COURSE

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ABSTRACT

Why do some students do well in College Algebra, but poorly in Trigonometry? Reasons include the array of new mathematical symbols, after years of working with the same symbols (+, −, ×, ÷), and the belief that trigonometry is unrelated to College Algebra and Calculus. Some students consider trigonometry to be a weed-out math course that expedites failure of Calculus courses and that their real downfall in mathematics is due to Trigonometry. Some calculus II students complain that: "As soon as I see a unit circle, or bizarre symbols such as sin θ or cos θ, my mind freezes" and "I failed your calculus course because I hate trig, otherwise I had no problems with the calculus." This paper tries to mitigate the students' fears about trigonometry by presenting it like college algebra. The ideas presented will seamlessly introduce algebra students to trigonometry.

Keywords: Trigonometry, Pythagorean Triples, Vedic mathematics.

INTRODUCTION

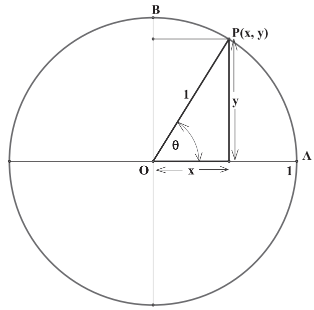
Teaching a Trigonometry course is a daunting task, especially at the collegiate level, since professors have only half the time to cover the material as the teachers in high schools. However, Trigonometry is such a beautiful and interlinked subject that the authors believe it is one of the easiest math courses to teach as a core curriculum course, as we will demonstrate below. Our motivation to write this paper is to show students that trigonometry is not a foreign object. Many students give up on a calculus problem if the problem contains any trigonometry in it. "I hate trigonometry"; "I took it two semesters ago. I hated it then and I hate it even more now"; "I cannot prove a single trigonometric identity even though I want to become a mechanical engineer"; and the list goes on and on. Students complain that they failed their calculus course because of trigonometry. Knowing that students have this mindset we should encourage them to get interested in, and not be afraid of learning new symbols.

Trigonometry.

Trigonometry has many applications and has numerous interconnections with other subjects. It is one of the most applicable mathematics courses and it is utilized in Physics, Engineering, Chemistry, Aeronautics and much more. Trigonometry is a subject, interconnected and application oriented, that extensively uses College Algebra. We present this paper to show how closely College Algebra and Trigonometry are interconnected and *how we could teach trigo-*

nometry and a college algebra course in a similar fashion. We will do this by elaborating on some theorems.

THEOREM 1: The Pythagorean Trigonometric Identities are the same as the equation of a unit circle.



Let us consider the unit circle $x^2 + y^2 = 1$

Let θ be any angle as shown. Then:

$\frac{x}{1} = \cos\theta \rightarrow x = \cos\theta$ and

$\frac{y}{1} = \sin\theta \rightarrow y = \sin\theta$. Thus

$\tan\theta = \frac{y}{x}$

The equation of the unit circle is $x^2 + y^2 = 1$ and using the equations above, we get:

(1) $\cos^2\theta + \sin^2\theta = 1$

(2) $1 + \tan^2\theta = \sec^2\theta$. *Dividing equation (1) by* $\cos^2\theta$

(3) $\cot^2\theta + 1 = \csc^2\theta$. *Dividing equation (1) by* $\sin^2\theta$

Thus three Pythagorean Trigonometric Identities and the equation of the unit circle are the same.

Remark: The students should realize that the Pythagorean trigonometric identities are *one and the same identity*. Students should practice to verify this and should get to a point that they would not forget them and use them spontaneously. Unfortunately it is true that even in a Calculus II course students do not remember them or do not know how to use them. We believe that the common problem among students is probably the time they spend to understand the identities.

The basic premise behind the identities is *the Unit Circle*. To our surprise, even some students in a set theory course could not tell us the equation of a unit circle. Students should be serious about learning and make learning their first priority. We believe that making students' *primary and secondary* curriculum much stronger would be a step in the right direction to fixing this problem.

TRIGONOMETRY IDENTITIES EXAMPLES

We will now present some examples of *trigonometry identity* problems and show how to solve them as *college algebra* problems.

Example 1: Verify $\cot\theta + \tan\theta = \sec\theta \bullet \csc\theta$

We begin with the left hand side **(LHS)** of the equation and recall that $\cot\theta = \frac{\cos\theta}{\sin\theta}$. We know that for any angle θ, $\cos\theta = x$ in the unit circle. Likewise $\sin\theta = y$.

$\therefore\ \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{x}{y}$. Similarly we have $\tan\theta = \frac{y}{x}$.

We thus get for the LHS: $\frac{x}{y} + \frac{y}{x} = \frac{x \cdot x}{x \cdot y} + \frac{y \cdot y}{x \cdot y} = \frac{x^2 + y^2}{x \cdot y} = \frac{1}{x \cdot y} = \frac{1}{x} \cdot \frac{1}{y}$ = $\sec\theta \bullet \csc\theta$

If we consider the right hand side **(RHS)** of the identity, we see that it is equal to what we found for the LHS, namely: $\sec\theta \bullet \csc\theta$.

The basic idea is to change the 'new' symbols (secθ, cscθ, tanθ, cotθ, …) back to the algebraic symbols and coordinates x and y in the unit circle. Now we have familiar symbols that our students are used to, and then solve the identity as an algebraic problem.

Example 2: Verify: $\frac{\cot\theta^2}{1+\csc\theta} = \frac{1-\sin\theta}{\sin\theta}$.

We start with the LHS of the identity replacing cotθ, cscθ and sinθ by coordinates in the unit circle: **LHS** = $\frac{\frac{x^2}{y^2}}{1+\frac{1}{y}}$. We then multiply the numerator and the denominator by y^2 and get: $\frac{\frac{x^2}{y^2}\cdot y^2}{\left(1+\frac{1}{y}\right)\cdot y^2} = \frac{x^2}{y^2+y}$. Since $x^2 + y^2 = 1$ it follows that $x^2 = 1 - y^2$. Therefore, the **RHS** of the equation becomes: $\frac{1-y^2}{y\,(y+1)}$ after factoring the *denominator*.

The numerator $1 - y^2$ can also be factored and becomes $(1 + y)(1 - y)$.

So: $\frac{x^2}{y^2+y} = \frac{(1+y)(1-y)}{y(y+1)}$

After reducing the rational form to simplest terms we find that:

$\frac{(1+y)(1-y)}{y(y+1)} = \frac{(1-y)}{y} = \frac{1-\sin\theta}{\sin\theta}$ = **RHS**

Once again we solved the problem in example 2 as an exercise in algebra so that we did not have to deal with unknown symbols.

Example 3: Verify: $\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B-\sin A\sin B} = \frac{\tan A+\tan B}{1-\tan A\tan B}$

Beginning with the LHS of the identity we need to prove, we remind the students that *cos A* = x_1; *cos B* = x_2 and similarly for *sin A* we write y_1 and for *sin B:* y_2

The LHS of the identity is then $\frac{x_1y_2+y_1x_2}{x_1x_2-y_1y_2}$.

We also would like to point out that if we can get the left hand corner of the denominator equal to 1, we make a small step forward. We do that by dividing numerator and denominator by x_1x_2.

We then obtain: $\frac{x_1y_2+y_1x_2}{x_1x_2-y_1y_2} = \frac{\frac{y_2}{x_2}+\frac{y_1}{x_1}}{1-\frac{y_1y_2}{x_1x_2}} = \frac{\tan B+\tan A}{1-\tan A\cdot\tan B}$ = RHS

The students need to be reminded that $x_1^{\,2} + y_1^{\,2} = 1$ and $x_2^{\,2} + y_2^{\,2} = 1$ since (x_1,y_1) and (x_2,y_2) are points on the unit circle.

Example 4:

Verify: $\dfrac{\sin^3\beta + \cos^3\beta}{\sin\beta + \cos\beta} = 1 - \sin\beta\cos\beta$

$$LHS = \frac{y^3 + x^3}{y + x}$$

$$= \frac{\cancel{(y+x)}(y^2 - yx + x^2)}{\cancel{(y+x)}}$$

$$= (y^2 + x^2) - yx$$

$$= 1 - \sin\beta\cos\beta = \text{RHS}$$

Remark: These examples should depict to our students that trigonometry is not a foreign object. They might become aware that it is in fact algebra, up to Pythagorean identities, dealing with sines and cosines that translate into x and y coordinates. It may alleviate the fear of students who hate trigonometry. The unit circle may seem friendlier. The authors believe that for a majority of our students this work should be refreshing. Some students may still like the older way better as change is harder to adapt to.

We will now consider an example that is more involved.

Example 5: Verify $\tan 2t = \dfrac{2\tan t}{2 - \sec^2 t}$

Based on our experience with the course this example seems to ask for a lot more from the students than the previous examples. The first question we have to ask is: how do we find sin 2t, cos 2t and tan 2t? In order to do so we first work with Pythagorean triples to obtain their sum and difference.

THEOREM 2: Given two Pythagorean triples, we can obtain two new Pythagorean triples by "adding" in the Vedic style or "subtracting" two given Pythagorean triples to obtain new Pythagorean triples.

Proof: Students had valid questions when the form with cos 2t and sin 2t appeared and they wondered what to do next. For this we would use the Pythagorean triples. We propose an elegant solution by adding two Pythagorean triples to obtain a new Pythagorean triple. For example, how do we add the triples 12, 5, 13 and 4, 3, 5, to get a new Pythagorean triple? First of all we explain the formula known as *"Vertical and Crosswise."* This formula originated in Vedic mathematics (mathematics derived from Hindus' sacred Scriptures called "Vedas"). We begin with two sets of any three numbers ***(not necessarily Pythagorean triples)*** and produce a third row.

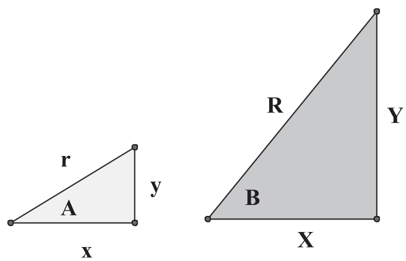
For example:

$$\begin{array}{ccc} 7 & 5 & 4 \\ 3 & 6 & 2 \\ \hline -9 & 57 & 8 \end{array}$$

We use the vertical and crosswise formula to obtain the three numbers in the third row: the first of the three new numbers, - 9, is obtained by *multiplying vertically the first two* numbers 7 x 3 and 5 x 6, and then by taking their *difference* (21 – 30 = -9.) To obtain the second number, 57, we *multiply crosswise* the numbers in the first two columns and add them (7 x 6 + 5 x 3 = 42 + 15 = 57). To obtain the third number, 8, we *multiply the last two numbers vertically* (4 x 2 = 8). The question is if this simple process helps us to generate new Pythagorean triples. At first glance it appears to be a hoax. The students had no idea where we were going.

Vedic Mathematics. We use *"Vedic"* mathematics to "add" two Pythagorean triples to find a *new* Pythagorean triple. Vedic Mathematics was used in India thousands of years ago and it was discovered from the Vedas (The main Hindu Religious Scriptures). One of the authors learned some of it in high school; his math teacher was very fond of Vedic mathematics. Vedic mathematics was rediscovered from the Vedas between 1911 and 1918 by Sri Bharati Krsna Tirthaji (1884 – 1960) (1). According to his research, all of mathematics is based on only sixteen Sutras or word-formulae. It does not seem plausible to the authors in today's environment that all the progress in mathematics derives from sixteen formulas. One of the authors read a few pages from the text "TRIPLES" by Kenneth Williams (2), and it refreshed some of his childhood memories. He also consulted the website www.hinduism.co.za/vedic.htm and read chapters from the text "Vertically and Crosswise" (3). Now we will show how the ancient methods from India can help us understand more trigonometry and geometry.

Adding two Pythagorean triples: Let A: (x, y, r) and B: (X, Y, R) be two Pythagorean triples (right triangle sides that are integers) shown below in the two triangles.



The new Pythagorean triple, the "sum", is then obtained as follows (exactly similar to the example shown previously):

Note that the difference in the cos column is taken to be positive.

angle	cos	sin	hypotenuse
A	x	y	r
B	X	Y	R
A+B	xX-yY	xY+yX	rR

Proof: $(xX - yY)^2 + (xY + yX)^2 = x^2X^2 + y^2Y^2 + x^2Y^2 + y^2X^2 + (2\,x\,y\,XY - 2\,x\,y\,XY) = x^2(X^2 + Y^2) + y^2(X^2 + Y^2) = (x^2 + y^2) \bullet R^2 = r^2R^2$.

It shows that adding *two Pythagorean triples* gives us a *new* Pythagorean triple. One needs to be careful that it is *not vertical* adding of numbers. Nor is it a determinant from matrix theory!

As a byproduct, we obtained the following two trig identities:
cos (A + B) = x X − y Y = cos (A) cos (B) − sin(A) sin (B) and
sin (A + B) = x Y + y X = sin(A) cos (B) + cos (A) sin (B).

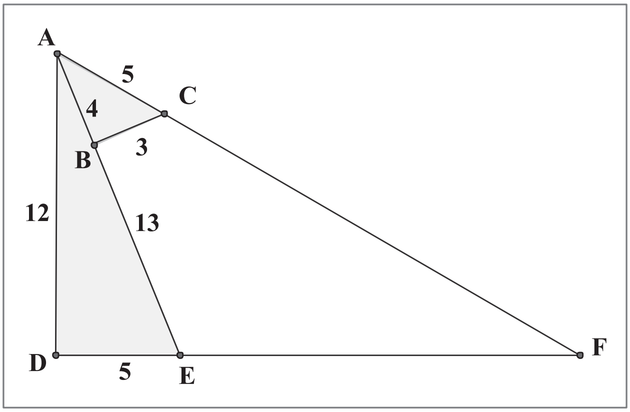
Note: Before we get the cosine or sine, the Pythagorean numbers x, y, r and X, Y, and R need to be reduced to $\frac{x}{r}$ for the cosine and $\frac{y}{r}$ for the sine, becoming rational numbers.

Example 6: Creating New Pythagorean Triples

?	**cos ?**	**sin ?**	
A	12	5	13
B	4	3	5
A+B	(48-15) 33	(36+20) 56	65 65

Observe that $33^2 + 56^2 = 65^2$ or 1089 + 3136 = 4225. *Thus "adding" the Pythagorean triples (12, 5, 13) and (4, 3, 5), using vertically and crosswise generated numbers produces another Pythagorean triple (33, 56, 65).*

In the diagram below we show the two triangles in the "Sum" position. Notice that the larger triangle is not yet the triangle with Pythagorean integers. The sides of this triangle however are rational numbers 12, $20\frac{4}{11}$ and $23\frac{7}{11}$. To find the Pythagorean triple one needs to multiply these rational numbers by a factor of $2\frac{3}{4}$ to get 33, 56 and 65.



For the identity in Example 5 above, we need to find *sin 2A, cos2A*, and *tan 2A*. We first need to find the "sum" corresponding to the use of the same triple, twice. Using the sum formula that we obtained above we find:

?	cos ?	sin ?	
A	x	y	r
A	x	y	r
2A	$(x^2\text{-}y^2)$	(2xy)	r^2

In particular for a *unit* circle

?	cos ?	sin ?	
A	x	y	1
A	x	y	1
2A	$(x^2\text{-}y^2)$	(2xy)	1

For the sake of completeness and clarity for students we show that both quantities from our table, $(x^2 - y^2)$ and (2xy), will represent in this unit circle context, actual trigonometric quantities that satisfy the requirement that they are X and Y- coordinates taken from the unit circle. To prove that (and re-connect to the unit circle) we show that $[\cos(2A)]^2 + [\sin(2A)]^2 = 1$!

Proof: x and y are on the unit circle so $x^2 + y^2 = 1$. If we square the cosine and the sine of 2A we find:

$(x^2 - y^2)^2 + (2xy)^2 = x^4 + y^4 - 2x^2y^2 + 4x^2y^2 =$

$x^4 + 2x^2y^2 + y^4 = (x^2 + y^2)^2 = 1^2 = 1$

Note that it proves the trig identities:

$$\begin{bmatrix} \cos 2A = x^2 - y^2 = cos^2 A - sin^2 A \\ \sin 2A = 2xy = 2\sin A\cos A \\ \tan 2A = \frac{2xy}{x^2 - y^2} \end{bmatrix}$$

We now return to Example 5 stated above: verify that $\tan 2t = \frac{2tant}{2 - sec^2 t}$. After using the identities from above we find:

LHS: *tan2t* $= \frac{2xy}{x^2 - y^2}$ *which equals* $x^2 + x^2 - y^2 - x^2 = 2x^2 - (x^2 + y^2)$ (subtract x^2 and add it at the same time) $= \frac{2xy}{2x^2 - (x^2 + y^2)} = \frac{2xy}{x^2 - y^2}$.

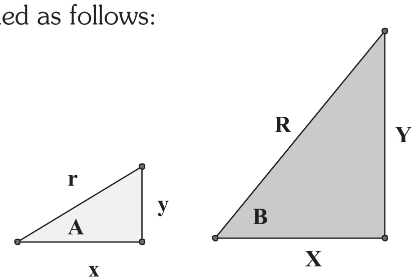
Dividing both top and bottom by x^2 we obtain:

$$\frac{\frac{2y}{x}}{2 - 1 - \left(\frac{y}{x}\right)^2} = \frac{2\tan t}{2 - (1 + \tan^2 t)} = \frac{2\tan t}{2 - \sec^2 t}$$

The work we have discussed so far begs the question: is the difference of two Pythagorean triples also a Pythagorean triple? Can we obtain it by using the “Vertical and Crosswise Sutra (rule)?” The answer is yes and is shown below.

Let A: (x, y, r) and B: (X, Y, R) be two Pythagorean triples. Then the new Pythagorean triplet (the difference) is obtained as follows:

?	cos ?	sin ?	
A	x	y	r
A	X	Y	R
A-B	(xX + yY)	(yX – xY)	rR



As in the case of the sum of the Pythagorean triples, the difference also gives the following identities:

cos(A - B) = xX + yY = cos (A) cos (B) + sin(A)sin(B) and

sin(A - B) = yX - xY = sin(A) cos(B) - cos(A)sin(B)

Example (notice the change of signs in the formulas!):

?	**cos ?**	**sin ?**	
A	12	5	13
B	4	3	5
A-B	(48+15) 63	(20-36) -16	65 65

Observe that $(63)^2 + (-16)^2 = (65)^2$ and that (3969) + (256) = 4225.

In the unit circle a negative value like -16 can be convertred into the co-ordinate $^{-16}/_{65}$ by dividing by the radius of 65 units. Thus by *subtracting* two Pythagorean triples, we obtain a *new* Pythagorean triple. Of course this is just an example and *not a proof*.

On the other hand, these ideas show that any two rows of triples that represent sides of a right triangle (even if they are NOT Pythagorean triples), produce a third row (a triple) that represents numbers that are still sides of right triangles!

Some advantages of studying Pythagorean triples and their connections to trigonometry:

- It helps students to understand the unit circle better, especially the fact that on a unit circle, $x = cos\theta$ *and* $y = sin\theta$.
- It is obviously simpler to work with.
- It shows a connection between algebra and trigonometry.
- Students need to know just one sutra (formula), "vertical and crosswise." With its help they can obtain many different trigonometric identities and thus they do not have to worry about memorizing them, which is one of the major complaints students have about trigonometry.
- This method can be used to solve trigonometric equations.

Remark: We think that in a classroom, it may be best to present the two methods *side-by-side*. The traditional method to prove trigonometric identities and the method(s) presented in this paper may help students to appreciate the connection between proving trigonometric identities and using algebra they have already learned.

We redo Example 1 using the traditional method and the method in this paper side by side.

Example 1 (revisited):

Verify: cot (θ) + tan (θ) = sec (θ) csc (θ). Begin with the left hand side:

LHS:

$\frac{x}{y}+\frac{y}{x}=$	$\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}=$
$\frac{x^2+y^2}{xy}$	$\frac{\cos\theta^2+\sin\theta^2}{\sin\theta\cdot\cos\theta}$
Using $x^2 + y^2 = 1$ we get:	Using $\cos^2\theta + \sin\theta^2 = 1$ we get:
$\frac{1}{xy}=$	$\frac{1}{\cos\theta\cdot\sin\theta}=$
$\frac{1}{x}\cdot\frac{1}{y}=$	$\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}=$
$\sec\theta\cdot\csc\theta$	$\sec\theta\cdot\csc\theta$

Comparing the two solutions one realizes that they are identical, and thus we can repeat the solutions of all the remaining four examples using two columns. The next obvious question is: can we solve *trigonometric* equations using this method?

We try a couple of examples:

Example A: Solve $\tan^2\theta = 5 + \sec\theta$. Replace the trigonometric symbols with coordinates in the unit circle: $\frac{y^2}{x^2} = 5 + \frac{1}{x}$. Notice that the equation contains x and y and that all this happens on the unit circle with equation $x^2 + y^2 = 1$. So if we replace y^2 by $1 - x^2$ we get the following equation: $\frac{1-x^2}{x^2} = 5 + \frac{1}{x}$.
Multiply both sides by $x^2 \rightarrow 1 - x^2 = 5x^2 + x$. Or: $6x^2 + x - 1 = 0$.

or $(3x - 1)(2x + 1) = 0$

or $x = \frac{1}{3}$ or $x = -\frac{1}{2}$

or $\cos\theta = \frac{1}{3}$ or $\cos\theta = -\frac{1}{2}$

$\theta = 1.23 \pm 2n\pi$, where n is any integer or $\theta = \frac{2\pi}{3} \pm 2n\pi$, where n is any integer

Example B: Solve the equation

$\cos(2\theta) + 3 = \sin\theta$

$(1 - 2\sin^2\theta) + 3 = \sin\theta$

$2\sin^2\theta + \sin\theta - 4 = 0$

$2y^2 + y - 4 + 0$

So $y_{1,2} = \frac{-1\pm\sqrt{1+32}}{4}$

Since both values of y (= sin θ) are outside the interval [-1, + 1] this equation has no solutions.

At this point, we will show how more trigonometric *identities* can be derived using the identities we already had, using the "*vertically* and *crosswise*" method discussed above. We think *cos 2A* is a most versatile trigonometric identity and it is one of the easiest identities to remember. Most students remember the first Pythagorean trigonometric identity.

$$\cos^2 A + \sin^2 A = 1 \ldots\ldots(1)$$

Changing the "+" sign into a "–" sign in this identify, we obtain with minor changes the double angle identify: $\cos^2 A - \sin^2 A = \cos 2A$ ……(2)

Adding the two equations together we obtain:

$$2\cos^2 A - 1 = \cos 2A \ldots\ldots(3)$$

Subtracting the second from the first:

$$1 - 2\sin^2 A = \cos 2A \ldots\ldots(4)$$

thus $$\cos^2 A = \frac{1+\cos 2A}{2} \ldots\ldots(5)$$

and $$\sin^2 A = \frac{1-\cos 2A}{2} \ldots\ldots(6)$$

To obtain the half angle formulas (replace $2A$ by A and A by $\frac{A}{2}$) we obtain

$$\cos^2 \frac{A}{2} = \frac{1+\cos A}{2} \text{ and } \sin^2 \frac{A}{2} = \frac{1-\cos A}{2}$$

hence $$\cos \frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}} \ldots\ldots(7)$$

and $$\sin \frac{A}{2} = \sqrt{\frac{1-\sin A}{2}} \ldots\ldots(8)$$

The (±) signs are used based on the quadrant in which angle $\frac{A}{2}$ lies.

We now obtain the remaining identities using the sum and difference formula or trigonometric identities that we obtained earlier while using the vertical and crosswise formula to add/subtract Pythagorean triples. All the remaining identities are obtained by using these four. We have shown earlier how we obtain the double angle identities by putting $B = A$, that is by taking the angles to be equal. Given the double angle identity cos2A, we can obtain the half angle trig identities. The exciting thing is to see what happens if we replace the positive by a negative sign in the very first trig identity. This identity provides a professor with an *elegant idea* to demonstrate to students that in mathematics signs have a very important place, as noticed before. *Except for the sine and the cosine formula to solve triangles*, the remaining identities can be obtained by using the following identities. The addition and subtraction formulas are:

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

There are *four* Product to Sum formulas that can be obtained by taking the sum or difference of the above identities such as:

$\sin u \sin v = ½(\cos(u - v) - \cos(u + v))$

$\cos u \cos v = ½(\cos(u - v) + \cos(u + v))$

$\sin u \cos v = ½(\sin(u + v) + \sin(u - v))$

$\cos u \sin v = ½(\sin(u + v) - \sin(u - v))$

Substituting $(u + v) = c$ and $(u - v) = d$ to obtain $u = \frac{c+d}{2}$, $v = \frac{c-d}{2}$, the previous identities can then be re-written as:

$$\cos d - \cos c = 2\sin\left(\frac{c+d}{2}\right)\sin\left(\frac{c-d}{2}\right)$$

$$\cos d + \cos c = 2\cos\left(\frac{c+d}{2}\right)\cos\left(\frac{c-d}{2}\right)$$

$$\sin c + \sin d = 2\sin\left(\frac{c+d}{2}\right)\cos\left(\frac{c-d}{2}\right)$$

$$\sin c - \sin d = 2\cos\left(\frac{c+d}{2}\right)\sin\left(\frac{c-d}{2}\right)$$

$$\sin c - \sin d = 2\cos\left(\frac{u+v}{2}\right) sin\left(\frac{c-d}{2}\right)$$

This completes the proof of all the trig identities that are used in trigonometric courses *except for the sine and cosine formulas to solve a triangle*. The work is interesting and simple. Moreover, the students might realize that trigonometry is not a monster as many students believe it to be.

In this paper we used the formula "*vertically and crosswise*" to add or subtract Pythagorean triples. It is a simple and elegant formula to provide us with much trigonometry with a small effort. Moreover the sum and difference formulas for Pythagorean triples may themselves be published along with some nice applications of them. We reiterate that this paper contains everything that is taught in a trigonometric course except the exercise sets and trig applications, but of course it is not a trigonometric textbook. It also does not include angular and linear velocity formulas, the formulas to find the area of a triangle, sine and cosine formulas to solve a triangle and also applications that specifically use trigonometry. Lastly, it does not contain De Moivre's theorem.

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