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Patterns for Early Childhood Majors

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PATTERNS FOR EARLY CHILDHOOD MAJORS

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ABSTRACT

Nationwide students struggle in mathematics. Also many college students have not learned to find mathematics an interesting subject and as a consequence they find it very dry and boring. It is our responsibility to help them develop interest in learning mathematics. To assist them in developing strong roots in mathematics we should start early. We believe one of the ways to get them interested in math is to teach them using patterns.

Keywords: simple patterns, digit sums, problem solving, 100's chart, algebraic equations.

INTRODUCTION

Several authors have pointed out the importance of patterns. Bassarear (1) referred to patterns in the context of mathematics and stated that patterns *"…make mathematics more accessible to everyone."* According to Bennett and Nelson (2) *"Patterns play a major role in the solution of problems in all areas of life."* They furthermore state: *"Finding a pattern is such a useful problem-solving strategy in mathematics that some have called it the art of mathematics."*

One of the authors first discovered a mathematical pattern during an evening walk with his nine year old daughter when he was teaching her the multiplication tables for 19. She recognized spontaneously the patterns in the multiplication table of 19 (19, 38, 57, 76, 95, 114, 133, *etc*.) by observing that the products could be found by counting down the units digit by one and adding two's to the ten's place, as shown in the numbers listed above, *e.g.*, 19 x 5 = 95. Ever since, his interest in mathematical patterns has gradually increased and he has tried to rub it off onto his students in his mathematics education courses.

In this paper we present patterns that cover a spectrum of mathematics, ranging from elementary to lower division college mathematics courses. We cover patterns similar to the multiplication table of 19 mentioned above. The patterns can be taught from elementary to possibly middle grades level. We first define a concept (better: a function) called "Digit Sums". For example, the first digit sum of (735) is found as follows: add 7 + 3 + 5 = 15. This represents the First Digits Sum. We then do the same for 15 = 1 + 5 = 6 to find the Second Digit sum. The unit digit '6' is the Final Digit Sum of 735, because we have reached one single digit.

THE HUNDRED'S CHART

Let us consider the following hundred's chart, usually introduced to elementary level students. We would like to know if this chart contains any patterns. First we notice some simple patterns in the rows and columns of the chart, but gradually we realize that this chart is a goldmine of patterns that can be understood by students at various academic levels. This motivated us to look for even more patterns in the numbers chart.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 1. Numbers chart.

To begin with, each row and each vertical column contains ten numbers. In each row any two consecutive numbers differ by 'one,' and in each column any two consecutive numbers differ by ten. Along the diagonals from *right* to *left* in the chart we notice for example that the numbers along a typical diagonal for example the one that begins at 5, the numbers from right to left are: 5, 14, 23, 32, and 41. This diagonal begins at *five*, contains *five* numbers and the digit sum of each number is also *five*, that is (1 + 4 = 5, 2 + 3 = 5, 3 + 2 = 5, and 4 + 1 = 5). Finally, the difference between any two consecutive numbers of the diagonal 5, 14, 23, 32 and 41 is *nine*. It should be pretty exciting for elementary grade level students. We believe this could help engage students' curiosity to find out what happens along other diagonals (from right to left) in the chart. And if a teacher would turn his students loose at this point to investigate what happens to the numbers in other diagonals from *left* to *right* and *right* to *left*, they would find that similar properties hold along all the other diagonals.

We focus on the diagonals from right to left beginning at *eight* (8, 17, 26, 35, 44, 53, 62 and 71) and one beginning with *thirty* (30, 39, 48, 57, 66, 75, 84, and 93). The diagonal that begins with the digit *eight*, contains *eight* numbers and the Digit Sum of each number in the diagonal is *eight* (1 + 7 = 8; 2 + 6 = 8, 5 + 3 = 8, *etc*. all equal to the first number in the diagonal). Moreover the difference between any two consecutive numbers in this diagonal is nine.

The diagonal beginning with thirty (30, 39, 48, 57, 66, 75, 84, and 93) does not seem to follow the same patterns as above. However, considering it as a diagonal that begins with 3 + 0 = 3 (the Digit Sum) instead of a diagonal beginning with 30 we note that the *digit sum* of each number in the diagonal is 3, (that is 3 + 9 = 12; and finally 1 + 2 = 3), and similarly for each of the numbers in the diagonal the *final digit sum* is *three*. Also the difference between any two consecutive numbers of the diagonal is *nine*. We do notice an anomaly in this diagonal: it contains neither three numbers nor thirty numbers as was the case with diagonals of the numbers discussed above.

The diagonal beginning with *ten* (10, 19, 28, 37, 46, 55, 64, 73, 82, 91) contains exactly ten numbers, with the *final digit sum* of each number being *one*, and the difference between any two consecutive terms of the diagonal being *nine*. All the diagonals above this main diagonal (beginning at 10) follow the same suit as the main diagonal, however all the diagonals below the main diagonal have the anomaly that the number of elements in these diagonal differs from the beginning number.

On a separate note and of special interest is the diagonal beginning at *nine*. The numbers along the diagonal (right to left) beginning with nine are: 9, 18, 27, 36, 45, … 81. They satisfy all the properties mentioned in the previous examples. However the numbers along this diagonal have a unique property: the numbers 9, 18, 27, 36, 45, 54, 63, 72, 81 form a *partial* multiplication table of nine (9 x 1 … to 9 x 9).

Another interesting property exists for numbers in each of the ten *columns*. In the first *column*, the First Digit Sums are: 1+1=2, 2+1=3,…, 9+1=10 (for this last sum we stop at 10). Similarly, the first digit sums of the ten numbers in the second column are 2, 3… 11. And finally in the ninth column, the digit sums are 9, 10, 11, …, 18. And in the tenth column, the digit sums of the 10 numbers are again 1, 2,…, 10 which is the same as in the first column. At this point a teacher may ask her students what they think about the digit sum of the numbers in the eleventh column if it (the column) was there. And how would the other sums change? After considering columns, one may wonder what about the rows? The rows depict similar properties if we avoid the tenth column. And if the table above begins at zero instead of one what difference would it make to the properties discussed above such as digit sums or number of elements in diagonals?

PATTERNS FOR DIAGONALS

The patterns for diagonals from left to right depict equally nice patterns. The longest diagonal is 1, 12, 23, 34, 45, 56, 67, 78, 89, and 100. We notice that the First Digit Sums of the numbers along this *diagonal* (omitting the last numeral 100) are 1, 1 + 2 = 3, 2 + 3 = 5, 3 + 4 = 7, … ,8 + 9 = 17. At this point teachers can pose a query to their class to analyze this pattern and have the students share their opinion with the class. This is a nice question for students to work out together in groups, which could help them learn from each other, depending on the structure of the class. We note that each consecutive pair of numbers in the list 1, 3, …, 17 differs by 2. We next check to see if something similar holds for numbers in the diagonals from left to right? Note that any two consecutive numbers in the main diagonal 1, 12, 23,… , 89, 100 differ from each other by *eleven*.

We go now to the diagonal starting with 21, going from left to right: 21, 32, 43, 54, 65, 76, 87, 98, one should observe first that any two consecutive numbers in this diagonal differ by *eleven*, and the first *digit sums* are 3, 5, 7, 9, 11, 13, 15, and17, once again each consecutive pair in these numbers differ by *two* the same as above. Let us check another diagonal, above the main diagonal: 3, 14, 25, 36, 47, 58, 69, and 80. First we note that any two consecutive numbers in this diagonal differ by *eleven*. Moreover, the *digit sums* are 3, 5, 7, 9, 11, 13, 15; and the difference between any two consecutive numbers is *two*. Since the number *eighty* does not contribute to the sequence 3, 5, …, 15, we decided to ignore it.

Referring to the 100's chart for reference, the diagonal from left to right beginning at 11 is of *special* interest. The numbers along this diagonal are 11, 22, 33, 44, 55, 66, 77, 88, and 99. Of course the difference between any two consecutive numbers is *eleven*. And in the sequence of *digit sums*: 2, 4, 6, 8, 10, 12, 14, 16, and 18, any two consecutive numbers differ by two, which is also a multiplication table of 2. Finally, the numbers 11, 22, 33, 44, …, 88, 99 give a partial multiplication table of *eleven* (11 x 1 = 11, 11 x 2 = 22, … , 11 x 9 = 99).

The diagonals from left to right have *another nice property*: If instead of taking DIGIT SUMS of the elements along the diagonal, we take DIGIT DIFFERENCES, (larger digit – smaller digit), we get the same digit for all the numbers along that diagonal. For example, the diagonal beginning at 6: 6, 17, 28, 39, 50, the difference between the digits of any two numbers is (6 - 0 = 6, 7 - 1 = 6, … , 9 – 3 = 6), As before, we discard number 50 as it does not contribute to the same difference. Similarly, let us consider the diagonal 61, 72, 83, 94, the digit differences in each of the four numbers is 5. Students can investigate on their own what happens to diagonals further on.

DEVELOPING ALGEBRAIC EQUATIONS

Another property we observed is that if we double the entries in any row, and subtract from it the corresponding entries in the following row, we then get the corresponding entries of the previous row. For example: we double the entries in the *second* row to get 22, 24, 26… 38, 40 and from it we subtract the corresponding entries in the *third* row 21, 22, 23, … , 29, and 30, then we get the corresponding entries 1 … 10 of the first row. It works for each and every row from row two to row nine. The natural question to ask here is what it is good for. The idea is if we could help students to translate the property into a simple algebraic equation, the idea would be worth our while. Let the phrase "row n" denote entries in any row from n = 2 to n = 9 and let it be clear that" row n" means all the entries in that row. We next multiply row n by 2 giving "row 2n," and now from these entries, we subtract the corresponding entries of the row 'n + 1,' and finally check to see if we got the entries of the row 'n – 1.' Thus algebraically we can help these elementary level students to write an equation (or an identity): $2n - (n + 1) = (n - 1)$, a *neat* idea! Students need to be reminded that (n – 1) does not mean the natural number 'n-1' but in fact it means all the numbers in the row.

This activity could motivate students to investigate if a similar idea works for the numbers in columns. It does, for columns 2 through 9. Hopefully, this activity may help students to perceive mathematics as an interesting subject. We provide one more activity along the same line and hope that students may try many more ideas along similar lines of inquiry. We multiply the numbers in row two by three and then subtract from it the corresponding number of row three, and we find: 12, 14, 16, 18, 20, 22, 24, 26, 28, and 30. Note that these numbers are not from any row; however, they did result in a nice pattern, even numbers from 12 to 30. This haphazard try may not result in something that we may be looking for. We try to understand what is happening algebraically. First multiply each number of a row 'n' by 3 and then subtract the corresponding numbers of row 'n + 1' from it that is to obtain (3n – (n+1)). This results in '2n – 1.' We note that '2n' may not be any row and as such '2n – 1' may not be any row. The next question arises: are the numbers '2n – 1' the same as 12, 14, ... 30. Unfortunately, the answer is no! Thus in this case we cannot even form an algebraic equation. The moral of the story is that students and teachers need to be careful; the patterns require some serious work.

SUMMARY

In this paper we presented various patterns derived from a hundred's chart. We argued that within rows, columns and diagonals, multiple regularities or patterns can be found. If we look deeper into these rows, columns or diagonals and we apply notions like the First sum of digits, and Final sums of digits, more regularities emerged. We introduced some algebra into our investigations to generalize what we noticed. These spontaneous investigations can open a person's mind to the many hidden connections among numbers. We also showed that serious algebraic study was required to make the case for claims about properties and to establish the boundaries of those claims.

REFERENCES

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