Georgia Journal of Science

Volume 71 *No. 2 Scholarly Contributions from the Membership and Others* Article 11

2013

Connecting Algebra and Geometry for Extraneous Roots

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CONNECTING ALGEBRA AND GEOMETRY FOR EXTRANEOUS ROOTS

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ABSTRACT

Extraneous roots are an unsolved mystery for our freshmen level college math students. The students are often confused and wonder what extraneous roots are and how do they suddenly appear while solving an algebraic equation. In this paper we clarify the idea of extraneous roots and connect the algebra and geometry behind extraneous roots.

Key words: Extraneous roots, Geometry of Extraneous Roots, x-Intercepts

INTRODUCTION

Most high school students and college freshmen struggle with the concept of extraneous roots in an algebra course (1). They often wonder why they have to check their answer in the original equation, especially when they square both sides of an equation containing radicals. In this paper, we first present a simple algebraic idea to explain why squaring an equation may introduce extraneous roots. We then illustrate the geometry involved behind this idea. Finally, we connect the equation back to algebra using one-to-one functions.

Let us consider the problem of solving the equation $x - 2 = 0$. We know that $x = 2$ is the root of this equation. Rewriting this equation as $x = 2$ and squaring both sides, we obtain the equation $x^2 = 4$. This equation has two roots namely $x = 2$, and $x = -2$. This is a basic example illustrating how squaring both sides of an equation may introduce extraneous roots.

The above example raises a question: does squaring both sides of an equation always interject extraneous roots? The answer to this question is no, not always. Consider the equation $\sqrt{2x-1} = x$. Squaring both sides of this equation and simplifying results in, $x^2 - 2x + 1 = 0$ and $x = 1$. Thus, in this example, squaring both sides of the equation did not introduce an extraneous root. This result raises another question: does squaring an equation containing square roots ever introduce extraneous roots? Of course it does. Consider the equation $\sqrt{2x+1} = x - 1$. Squaring both sides of this equation and simplifying results in the equation, $x^2 - 4x = 0$. This equation has two roots $x = 0$ *or* $x = 4$, where $x = 0$ is an extraneous root.

Look at the graphs of $y = x - 2$ and $y = x^2 - 4$ below. The equation $y = x - 2$ can be graphed by using the x-intercept of the line. The equation $x^2 = 4$ can be considered as the parabola $y = x^2 - 4$. Key points on the graph of the equation $y = x^2 - 4$ are the x-intercepts of this equation.

The line $y = x - 2$ has one x-intercept namely (2, 0) and it relates to the solution $x = 2$ for the *equation* $x - 2 = 0$

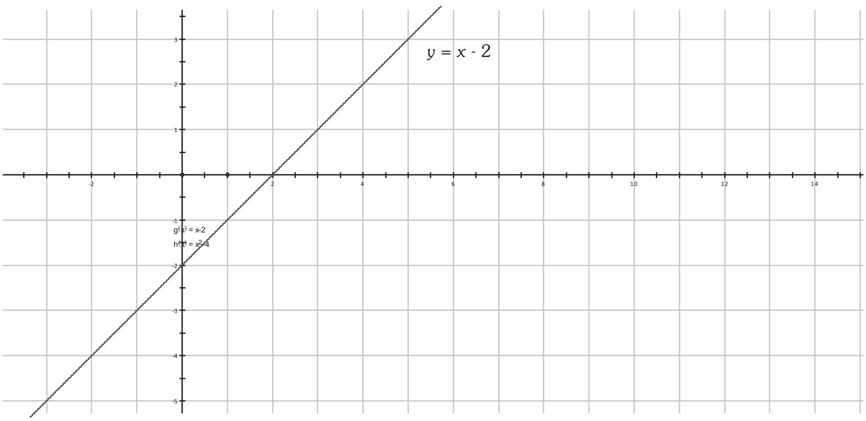

Figure 1. Before squaring the equation $x = 2$;

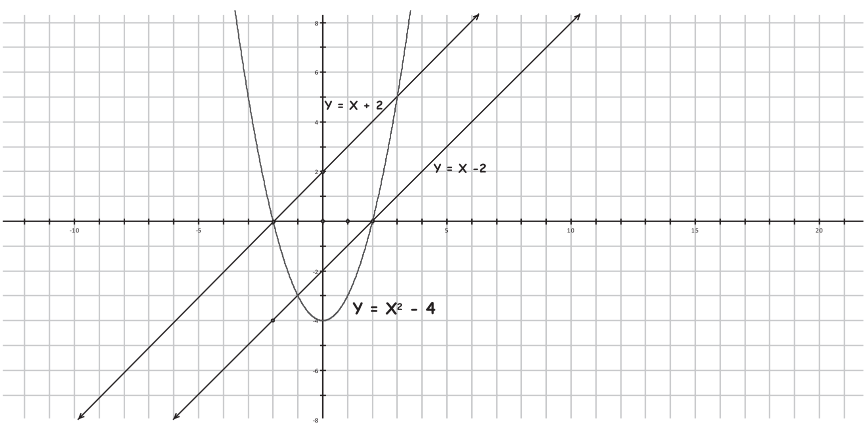

Figure 2. After squaring the equation $x = 2$ the x-intercepts of the parabola $y = x^2 - 4$ are $x = 2$ and $x = -2$ (see the text on the previous page).

The parabola $y = x^2 - 4$ has two x-intercepts, namely (-2, 0) and (2, 0), that correspond to the two roots $x = -2$, and $x = 2$ of the equation $x^2 = 4$.

Thus, geometrically we can see that squaring an equation changes the type of equation it is and thus may introduce extraneous roots.

Finally, returning to the algebra, the equation $f(x) = x - 2$ is a one-to-one function and thus for different x-values, the function values are different. However, $f(x) = x^2 - 4$ is not a one-to-one function and thus for different x-values we could have the same y-value; *e.g.*, in this case for both $x = -2$ and for $x = 2$, we have the same y-value, namely, $y = 0$. Similar arguments hold for the other two equations used in the above discussions.

Extraneous roots baffle our students whether it is an equation containing fractions or square roots etc. In this paper we have tried to connect algebra and geometry (2) with regard to extraneous roots, especially when both sides of an equation are squared. Showing connections in mathematics is significantly important and helps students to see mathematics as a whole. The Standards of the NCTM (3) and the focus in school mathematics in algebra emphasize teaching mathematics as a whole instead of teaching it as segregated branches and connecting algebra to geometry in a meaningful way.

REFERENCES

1. Wagner S P: "Advancing Algebra." Macmillan, In P.S. Wilson (Ed.), Research ideas for the classroom, High School Mathematics,119-139, 1993.
2. Graham K C and Zimmermann A G: "Focus in High School Mathematics: Reasoning and Sense Making in Algebra," Nat Coun Teach Math, 2009.
3. "Nat Coun Teach Math, Principles and Standards of School Mathematics," 2000.