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THE IMPORTANCE OF MATHEMATICAL WORDING
BY MATH INSTRUCTORS

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ABSTRACT
At times we the math instructors in a classroom are not careful enough to use proper Mathematical wording, e.g. \( \sin^2 + \cos^2 = 1 \) without any reference to the angle. Improper Mathematical wording can be confusing to a student at any level, especially at high school and/or at a college freshman or sophomore level. It may hinder the students’ progress in the present and the subsequent math courses.

Through our years of teaching, we have seen students struggle with many mathematical concepts. Oftentimes this difficulty does not arise with a lecture topic, but rather with the language and terminology that some of their previous teachers have used. (At times, most of us are guilty of this too.) In this paper we first discuss some examples of mathematical wording that can be confusing to students and then supplement it with worksheets in Appendix A that contain activities to substantiate the ideas discussed in the body of this paper. These activities can also be used in a classroom by an instructor as guided activities making students to do the work and the instructor to guide them along the way if and when help is needed by a student(s).

As our first example, when the concept of vertical asymptotes and the method of how to find them is introduced, an instructor may say “If an \( x \)-value makes the denominator of a rational function zero, then the function has a vertical asymptote at that \( x \)-value.” The teacher may forget to add that it is true ONLY when the rational function is in the reduced form. For example, the function \( y = \frac{x^2 - 9}{x^2 + x - 6} \) has only one vertical asymptote, \( x = 2 \), as it contains a removable discontinuity (a hole in its graph) at \( x = -3 \).

We have encountered many students who have had a difficult time in recognizing when the graph of a function has a hole instead of a vertical asymptote. This is compounded by the fact that they do not see a hole in the graph when they use a graphing calculator to graph the function. Moreover, they have become accustomed to the above language and expect a vertical asymptote at \( x = -3 \).

Another example is when we examine the special case of \( 0/0 \). As early as elementary school, students are told that “zero divided by any number is zero.” We think it is probably alright at that level, however, sometimes the
same explanation is continued by some of us even through the earlier years of the college curriculum. This presents a serious problem when students are introduced to the idea of limits; many struggle with the idea of division of zero being anything but zero, e.g. the \( \lim_{x \to 0} \frac{\sin x}{x} \) has to be zero. For many students, the correct answer “one” for this particular example is confusing and is “kind of unacceptable” (in their minds) as it contradicts what they have been learning all along. Also, students struggle with this idea when they are introduced to indeterminate forms of limits in calculus, as 0/0 can be looked upon in two different ways, one that zero is divided by a number, and the other that a number is divided by itself. Since most students remember that zero divided by any number is zero, and that a number divided by itself must equal one, the number 0/0 is totally confusing for many students.

As another example, looking at important components of trig identities, one can see how the omission of words can be misleading to students. For example, when stating trig identities, the angle is often omitted. The expression “\( \sin^2 \theta + \cos^2 \theta = 1 \)” is often stated “sine squared plus cosine squared equals one.” This can lead students to think that the angle is not important and that the angle for the sine function and the angle for the cosine function need not agree for the identity to be true. Of course, this is not the case.

We can list some other examples. However, we are just trying to make a point that a math instructor needs to be careful of his/her choice of wording while teaching.

We now present guided activities to help students understand some of the concepts mentioned above. The first worksheet is to aid students to understand the difference between vertical asymptotes and holes (removable discontinuities) in the graph of a function. The first example on this worksheet reduces to a linear function. The graph of this rational function is a straight line with a hole in it, which is even more confusing for our students as the graph on a graphing calculator does not show either a vertical asymptote or a hole in the graph of the function. This example demonstrates that the graph of a rational function could be a straight line with hole(s) and may have no vertical asymptotes. It also helps students to better understand the domain of a function, especially for the functions that are written in the form, \( y = \frac{f(x)}{x - a}; \) (see the questions about it that we pose in the first worksheet.)

The second example in the worksheet is a rational function that reduces to another rational function with a hole in its graph. In addition to the ideas taught in example 1, this example helps students to understand that the graph of a rational function may not have a vertical asymptote for each value of x that makes the denominator equal to zero. Both examples also help students to realize that before graphing a rational function, it is necessary to put the function in a reduced form.
The second worksheet focuses on dividing by zero. The first example on this worksheet consists of a function in which both the numerator and the denominator approach zero as \( x \) approaches 2. (The discussion provided below assumes that students have not yet learned the concept of limit). Thus those students who remember that “zero divided by any number is zero” will guess that the function approaches zero as \( x \to 2 \). On the other hand students who remember that “a number divided by itself always equals one” will guess that the limit of the function is \( \frac{0}{0} \). The objective is to show that both answers are incorrect and we should be careful when we divide a zero by another zero.

If an instructor has already discussed the topic of evaluating limits of rational functions then most students may obtain a correct answer for this example. Thus, in this worksheet, we deliberately chose a second example that contains trigonometric functions. The question is, “what do you think happens to the function as \( x \) gets close to 0?” Students cannot reduce this function, and their guesses of “zero” or “one” are both incorrect answers. In this example the use of technology is quite helpful; using a graphing calculator, students can check that as \( x \) approaches zero, the function approaches \( \frac{1}{2} \). Of course many selective (bright) students can obtain the same answer by dividing each term in the numerator and the denominator by \( x \) and making use of the special limits \( \frac{\sin x}{x} \to 1 \) and \( \frac{\tan x}{x} \to 1 \) as \( x \to 0 \).

The third worksheet deals with omission of words especially in trigonometric identities such as \( \cos^2 + \sin^2 = 1 \) instead of \( \cos^2 \theta + \sin^2 \theta = 1 \) and thus it is self explanatory to the reader. Finally, we present the solutions to all examples on the three worksheets in the Appendix A.

To summarize, one of the standards of NCTM states, “all students can use the language of mathematics to express mathematical ideas precisely.” Every math instructor needs to carefully choose mathematical wording so as not to mislead students and hinder their progress in the current and subsequent courses. Hopefully, by making a proper choice of words in our classes, we could have more success in achieving this NCTM standard.

REFERENCES

Worksheet 1 – Rational Functions

1. Consider the function \( y = \frac{x^2 - 4}{x + 2} \).
   (a) What is the domain of this function?
   (b) Factor the function and write it in the reduced form.
   (c) What is the domain of the function in (b)?
   (d) Are the answers in (a) and (c) the same? Why or why not?
   (e) Based on your answer in part (d), are the functions going to have the same graph? If not, then what will be different?
   (f) Using your graphing calculator, graph the original function and the function in part (b).
   (g) Do the graphs “appear” to be the same? Does this agree with your answer in (e)?
   (h) What should happen for the graphs in (g) to agree with your answer in (e)?
      Make sure that you input your function in the proper form in your calculator, e.g., \( y = \frac{x^2 - 4}{x + 2} \) is an incorrect form.

2. Consider the function \( y = \frac{x^2 + 5x + 4}{x^2 + 3x + 2} \).
   (a) What is the domain of this function?
   (b) Factor the function and write it in the reduced form.
   (c) What is the domain of the function in (b)?
   (d) Are the answers in (a) and (c) the same? Why or why not?
   (e) Based on your answer in part (d), are the functions going to have the same graph? If not, then what will be different?
   (f) Using your graphing calculator, graph the original function and the function in part (b).
   (g) Do the graphs “appear” to be the same? Does this agree with your answer in (e)?
   (h) What should happen for the graphs in (g) to agree with your answer in (e)?
      Make sure that you input your function in the proper form in your calculator.
Worksheet 2 – Dividing by Zero

1. Consider the function \( y = \frac{x-2}{x^2-4} \).
   (a) What is the domain of this function? Is 2 in the domain of the function?
   (b) What happens to the numerator of the function as \( x \) gets close to 2?
   (c) What happens to the denominator of the function as \( x \) gets close to 2?
   (d) From your answer to parts (b) and (c), what do you think happens to the function as \( x \) gets close to 2?
   (e) Graph the function on your calculator and use the trace feature to see what happens to the value of the function as \( x \) gets closer to 2.
   (f) Are the answers to parts (d) and (e) the same?
   (g) Factor and simplify the function.
   (h) What is the domain of the function in part (g)?
   (i) In part (h), what value does the function have when \( x = 2 \)?

2. Consider the function \( y = \frac{\tan x}{x + \sin x} \).
   (a) What is the domain of this function?
   (b) What happens to the numerator of the function as \( x \) gets close to 0?
   (c) What happens to the denominator of the function as \( x \) gets close to 0?
   (d) From your answer to parts (b) and (c), what do you think happens to the function as \( x \) gets close to 0?
   (e) Graph the function on your calculator and use the trace feature to see what happens to the value of the function as \( x \) gets closer to 0.
   (f) Are your answers to parts (d) and (e) the same?
Worksheet 3 – Omitting Angles in Trigonometric Identities

1. Consider the identity $\sin^2 \theta + \cos^2 \theta = 1$. This is one of the three Pythagorean Identities of trigonometry.

   (a) Based on the above identity, fill in the blanks:
   
   i. $\sin^2 \psi + \cos^2 \psi = \ldots$
   
   ii. $\sin^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{4} = \ldots$
   
   iii. $\sin^2 \frac{2\pi}{3} + \cos^2 \frac{2\pi}{3} = \ldots$

   (b) Now compute the following:

   iv. $\sin^2 \frac{\pi}{2} + \cos^2 \frac{\pi}{2} = \ldots + \ldots = \ldots$

   v. $\sin^2 \frac{\pi}{6} + \cos^2 \frac{5\pi}{4} = \ldots + \ldots = \ldots$

   (c) Do your answers in part (b) contradict the identity $\sin^2 \theta + \cos^2 \theta = 1$? Why or why not?

2. Consider the identity $1 + \tan^2 \theta = \sec^2 \theta$. This is another of the three Pythagorean Identities of trigonometry.

   (a) Based on the above identity, fill in the blanks:

   i. $1 + \tan^2 \omega = \sec^2 \ldots$

   $1 + \tan^2 \frac{\pi}{4} = \sec^2 \ldots$

   ii. $1 + \ldots = \sec^2 \left(\frac{\pi}{3}\right)$

   iii. $1 + \tan^2 \ldots = \sec^2 \left(\frac{7\pi}{6}\right)$

   $1 + \tan^2 \ldots = \ldots$

   (b) Now compute the following:

   i. $1 + \tan^2 \frac{\pi}{4} = \sec^2 \ldots$

   $1 + \ldots = \ldots$

   ii. $1 + \tan^2 \frac{3\pi}{4} = \sec^2 \ldots$

   $1 + \ldots = \ldots$

   (c) Do your answers in part (b) contradict the identity $1 + \tan^2 \theta = \sec^2 \theta$? Why or why not?
Appendix A

Solutions for the questions in the Worksheet 1 – Rational Functions

1. Consider the function \( y = \frac{x^2 - 4}{x + 2} \).
   (a) What is the domain of this function? **All reals except** \( x = -2 \)
   (b) Factor the function and write it in the reduced form.
   \[
   y = \frac{(x - 2)(x + 2)}{x + 2} = x - 2
   \]
   (c) What is the domain of the function in (b)? **All reals**
   (d) Are the answers in (a) and (c) the same? Why or why not? **No,**
   **the domains do not match.**
   (e) Based on your answer in part (d), are the functions going to have
   the same graph? If not, then what will be different? **No, because
   at** \( x = -2 \), **the original function will have a hole; but the
   reduced function will not.**
   (f) Using your graphing calculator, graph the original function and
   the function in part (b).
   
   ![Original Graph](image1)
   ![Graph in (b)](image2)
   (g) Do the graphs “appear” to be the same? **Yes**
   Does this agree with your answer in (e)? **No**
   (h) What should happen for the graphs in (g) to agree with your an-
   swer in (e)? **There should be a hole at** \( x = -2 \) **in the graph
   of the original function.**

2. Consider the function \( y = \frac{x^2 + 5x + 4}{x^2 + 3x + 2} \).
   (d) What is the domain of this function? **All reals except** \( x = -1 \)
   **and** \( x = -2 \)
   (e) Factor the function and write it in the reduced form.
   \[
   y = \frac{(x + 1)(x + 5)}{(x + 2)(x + 1)} = \frac{x + 5}{x + 2}
   \]
   (f) What is the domain of the function in (b)? **All reals except** \( x = -2 \)
   (g) Are the answers in (a) and (c) the same? Why or why not? **No,**
   **they differ at** \( x = -1 \)
   (h) Based on your answer in part (d), are the functions going to have
   the same graph? **No**
   If not, then what will be different? **They will differ at** \( x = -1 \)
   (i) Using your graphing calculator, graph the original function and
   the function in part (b).
Original Graph

Graph in (b)

(j) Do the graphs “appear” to be the same? Yes. Does this agree with your answer in (e)? No

(k) What should happen for the graphs in (g) to agree with you answer in (e)? There should be a hole in the graph at \( x = -1 \)
Solutions for the questions in the Worksheet 2 – Dividing by Zero

1. Consider the function \( y = \frac{x-2}{x^2-4} \).

(a) What is the domain of this function? Is 2 in the domain of the function? All reals except \( x = -2 \) and 2; No

(b) What happens to the numerator of the function as \( x \) gets close to 2? It gets close to 0

(c) What happens to the denominator of the function as \( x \) gets close to 2? It gets close to 0

(d) From your answer to parts (b) and (c), what do you think happens to the function as \( x \) gets close to 2? It gets close to \((\text{zero/zero})\).

(e) Graph the function on your calculator and use the trace feature to see what happens to the value of the function as \( x \) gets closer to 2. The graph gets close to 0.25.

(f) Are the answers to parts (d) and (e) the same? No

(g) Factor and simplify the function. \( y = \frac{x-2}{x^2-4} = \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2} \)

(h) What is the domain of the function in part (g)? All reals except \( x = -2 \)

(i) In part (h), what value does the function have when \( x = 2? 0.25 \)

2. Consider the function \( y = \frac{\tan x}{x + \sin x} \).

(l) What is the domain of this function? All reals except \( x = 0 \)

(m) What happens to the numerator of the function as \( x \) gets close to 0? It gets close to 0

(n) What happens to the denominator of the function as \( x \) gets close to 0? It gets close to 0

(o) From your answer to parts (b) and (c), what do you think happens to the function as \( x \) gets close to 0? It gets close to \((\text{zero/zero})\).

(e) Graph the function on your calculator and use the trace feature to see what happens to the value of the function as \( x \) gets closer to 0. The function gets close to 0.5.

(q) Are your answers to parts (d) and (e) the same? No
Solutions for the questions in the Worksheet 3 – Omitting Angles in Trigonometric Identities

1. Consider the identity \( \sin^2 \theta + \cos^2 \theta = 1 \). This is one of the three Pythagorean Identities of trigonometry.

(a) Based on the above identity, fill in the blanks:
   i. \( \sin^2 \psi + \cos^2 \psi = 1 \)
   ii. \( \sin^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{4} = 1 \)
   iii. \( \sin^2 \frac{2\pi}{3} + \cos^2 \frac{2\pi}{3} = 1 \)

(b) Now compute the following:
   iv. \( \sin^2 \frac{\pi}{2} + \cos^2 \frac{\pi}{4} = 1^2 + \left( \frac{\sqrt{2}}{2} \right)^2 = \frac{3}{2} \)
   v. \( \sin^2 \frac{\pi}{6} + \cos^2 \frac{5\pi}{4} = \left( \frac{1}{2} \right)^2 + \left( -\frac{\sqrt{2}}{2} \right)^2 = \frac{3}{4} \)

(c) Do your answers in part (b) contradict the identity \( \sin^2 \theta + \cos^2 \theta = 1 \)? Why or why not? No, because there are different angles in the equations.

2. Consider the identity \( 1 + \tan^2 \theta = \sec^2 \theta \). This is another of the three Pythagorean Identities of trigonometry.

(a) Based on the above identity, fill in the blanks:
   i. \( 1 + \tan^2 \omega = \sec^2 \omega \)
   \[ 1 + \tan^2 \frac{\pi}{4} = \sec^2 \frac{\pi}{4} \]
   ii. \( 1 + \tan^2 \frac{\pi}{3} = \sec^2 \left( \frac{\pi}{3} \right) \)
   iii. \( 1 + \tan^2 \frac{7\pi}{6} = \sec^2 \frac{7\pi}{6} \)
   \[ 1 + \tan^2 \theta = \sec^2 \theta \]

(b) Now compute the following:
   i. \( 1 + \tan^2 \frac{\pi}{4} = \sec^2 \frac{5\pi}{4} \) No
   \[ 1 + (1)^2 = (2)^2 \]
   ii. \( 1 + \tan^2 \frac{3\pi}{4} = \sec^2 \frac{\pi}{2} \) No
   \[ 1 + (-1)^2 = \text{undefined} \]

(c) Do your answers in part (b) contradict the identity \( 1 + \tan^2 \theta = \sec^2 \theta \)? Why or why not? No, because the angles in each equation are different.