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INVESTIGATION OF A PEST CONTROL METHOD INVOLVING CHEMICAL AND BIOLOGICAL METHODS USING THE LOTKA-VOLTERRA MODEL WITH FISHING

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ABSTRACT

We are modeling two insect populations which have prey and predator relationship. If it is not controlled the prey can destroy agricultural fields. To prevent the outbreak of the prey population, pesticides are widely used. The pesticides kill the prey and may also kill the predator. We used the Lotka-Volterra model with fishing (pesticide is fishing agent) to model the prey-predator relationship, which is\[\begin{align*}
\frac{dX}{dt} &= (a - e - bY)X,
\frac{dY}{dt} &= (dX - c - f)Y.
\end{align*}\]
Our results suggest that, in the presence of the predator, the predation alone (without pesticide) can help to keep the prey population at low level. In some situations using pesticides can cause an increase of the prey population and it can cause also seasonal (periodic) pest (prey) outbreak.

INTRODUCTION

Agricultural pests (such as European corn borer, two spotted spider mite, rootworm... etc.) reduce crop yield and/or quality. For example, the cost to U.S farmers from the European corn borer (insect pest) is estimated to be over one billion U.S dollars per year (5). Chemical control methods (for example using synthetic pesticides such as DDT) are some of the most common pest management tools used today. With the discovery of DDT in 1941 chemical pest control was successful at first. However after a period of time, farmers encountered difficulties in connection with the use of pesticides. Many pests started to develop resistance to the chemicals and they could no longer be killed by that particular pesticide (5, 6, 7). Next, farmers started to see pests that they had never seen before in their fields, these pests were termed as secondary pests. The secondary pests (which were previously at very low population levels) occurred in large number because the pesticide used against another pest species killed the agents that kept it at low population levels (4). Overuse of pesticides cause problems such as pesticide resistance, secondary pest outbreaks, and environmental contamination.
For example, in 1970s, the increasing use of fertilizers and pesticides allowed farmers in Indonesia to have a large increase in rice production. Unfortunately, the extensive use of pesticides led to an enormous growth in the population of brown plant hoppers (insect pests). Scientists later showed that the extensive use of pesticide caused the outbreak of brown plant hoppers in the first place. The pesticide eliminated all the natural predators of the brown plant hoppers, particularly spiders (4).

To overcome the problem scientists have developed a pest management mechanism which is called Integrated Pest Management (IPM). IPM involves using a combination of cultural, biological, chemical and mechanical control methods. During destructive outbreaks, IPM considers the overall management of a pest species. The objective of IPM is to prevent pest outbreaks. Biological control utilizes natural enemies such as parasites, predators, pathogens or competitors, to reduce or eliminate pests on agricultural fields. It involves using a living organism to control the outbreak of harmful pests. It is acknowledged to be the best type of pest control.

Our research involves an investigation of the combination of chemical and biological pest control methods using a simple mathematical model. We assume that there are insect pests (prey) which attack the crop and their natural enemy ( predator) in the agricultural field. The pesticide is assumed to kill (certain percentage of the population) both the pests and their natural enemy. The model is a Lotka-Voltera model with fishing, where the fishing agent is the pesticide.

For parameter estimations we consider, particularly, the two spotted spider mite (insect pest) which feeds on a variety of plants (such as corn, soybean, etc.) and the fallacis mite which feeds ( predator) on the two spotted spider mite. Using data from the literature (1, 2), we estimated the growth rate (intrinsic increase of prey population), predation rate (decrease of prey population due to predation), intrinsic decrease rate of predator population and effect of predation on predators population's reproduction. Since the dose of the pesticide and its toxic level can be controlled, we assume that the (percentage) death rate due to the pesticide is a free parameter in the model (for both the two spotted spider mite and the fallacis mite). The purpose of the research is to study the dynamics of the two spotted spider mite (prey) and the fallacis mite ( predator) populations by taking all possible values of the free parameters. We analyzed the stability of the equilibrium values of the Lotka-Voltera with fishing model for all possible values of the free parameters.

**THE MODEL**

Let \( X(t) \) be the two spotted spider mite (prey) and \( Y(t) \) the fallacis mite population. The Lotka-Volterra predator-prey model, which includes fishing (pesticides), can be written as:
\[
\begin{align*}
\frac{dX}{dt} &= aX - bXY - eX \\
\frac{dY}{dt} &= -cY + dXY - fY
\end{align*}
\] (1.1)

Where \(a\) is intrinsic increase of prey population, \(b\) is predation rate (decrease rate of prey population due to predation), \(c\) is intrinsic decrease rate of predator population, \(d\) is effect of predation on predator population’s reproduction (increase rate of predator population due to predation), \(e\) decrease rate of prey population due to pesticide, and \(f\) decrease rate of predator population due to pesticide.

**The population dynamics of the prey in the absence of the predator population** \((Y(t)=0\) and \(X(t)>0)\)

If \(Y(t)=0\) then the equation (1.1) will be reduced to \(\frac{dX}{dt}=(a-e)X\), and the solution is \(X(t)=X_0e^{(a-e)t}\) where \(X_0\) is and initial population. If \(a>e\) then the prey population will grow exponentially. If \(a>e\) then \(\lim_{t \to \infty}X_0e^{(a-e)t}=0\) this implies that the prey population will eventually become extinct. Finally if \(a=e\) then \(X(t)=X_0\), which implies the prey will have a constant population size. Therefore, in the absence of the predator the prey population will explode. To control the outbreak (or to eliminate the pest) the pesticide has to be sprayed extensively to kill the pest at a rate higher than the intrinsic increase of prey population, that is \(e>a\).

**The population dynamics of the prey in the presence of the predator population** (when \(Y(t)>0\) and \(X(t)>0)\)

If \(X(t)>0\) and \(Y(t)>0\) then \(X_e>0\) and \(Y_e>0\) represent the equilibrium population of the prey and the predator respectively, and this equilibrium can be obtained by solving the right hand sides of equation (1.1):

\[
\begin{align*}
(aX_e-bX_eY_e-eX_e) &= 0 \\
(-cY_e+dX_eY_e-fY_e) &= 0
\end{align*}
\]

These equations can be factored to:

\[
\begin{align*}
((a-e)-bY_e)X_e &= 0 \\
((-c-f)+dX_e)Y_e &= 0
\end{align*}
\]

The first equation implies either \(X_e=0\) or \(Y_e=a-e\). Similarly, the second equation implies that either \(X_e=0\) or \(Y_e=c+f\). Thus, one of the equilibrium is \((X_e,Y_e)=(0,0)\) with neither the prey nor the predator existing. The other equilibrium satisfies \((X_e,Y_e)=\left(\frac{c+f}{d}, \frac{a-e}{b}\right)\) where the prey and the predator co-exist in the environment.
Local Stability Analysis

Local stability analysis is performed around an equilibrium point of the nonlinear differential equation \((1.1)\). If we linearise around the equilibrium point \((0,0)\), we obtain the Jacobian matrix:

\[
J_0 = \begin{bmatrix}
\frac{\partial(aX - bXY - eX)}{X}_{|_{(0,0)}} & \frac{\partial(aX - bXY - eX)}{X}_{|_{(0,0)}} \\
\frac{\partial(-cY + dXY - fY)}{Y}_{|_{(0,0)}} & \frac{\partial(-cY + dXY - fY)}{Y}_{|_{(0,0)}}
\end{bmatrix}
\]

\[
J_0 = \begin{bmatrix}
(a-bY-e)_0 & (-bX)_0 \\
(dY)_0 & (-c+dX-f)_0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
a-e & 0 \\
0 & -c-f
\end{bmatrix}
\]

\[
det\left(\begin{bmatrix}
a-e & 0 \\
0 & -c-f
\end{bmatrix} - \lambda \begin{bmatrix}
a & 0 \\
0 & b
\end{bmatrix}\right) = (a-e-\lambda)(-c-f-\lambda) = 0,
\]

which implies that 

\(J_0\) has eigenvalues \(\lambda_1 = a-e\) and \(\lambda_2 = -c-f\).

Similarly, if we linearise around the equilibrium point \(\left(\frac{c+f}{d}, \frac{a-e}{b}\right)\) then we obtain the Jacobian matrix:

\[
J_1 = \begin{bmatrix}
\frac{\partial(aX - bXY - eX)}{X}_{|_{\left(\frac{c+f}{d}, \frac{a-e}{b}\right)}} & \frac{\partial(aX - bXY - eX)}{X}_{|_{\left(\frac{c+f}{d}, \frac{a-e}{b}\right)}} \\
\frac{\partial(-cY + dXY - fY)}{Y}_{|_{\left(\frac{c+f}{d}, \frac{a-e}{b}\right)}} & \frac{\partial(-cY + dXY - fY)}{Y}_{|_{\left(\frac{c+f}{d}, \frac{a-e}{b}\right)}}
\end{bmatrix}
\]

\[
J_1 = \begin{bmatrix}
(a-bY-e)_{\left(\frac{c+f}{d}, \frac{a-e}{b}\right)} & (-bX)_{\left(\frac{c+f}{d}, \frac{a-e}{b}\right)} \\
(dY)_{\left(\frac{c+f}{d}, \frac{a-e}{b}\right)} & (-c+dX-f)_{\left(\frac{c+f}{d}, \frac{a-e}{b}\right)}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{-b(c+f)}{d} & 0 \\
\frac{d(a-e)}{b} & 0
\end{bmatrix}
\]

\[
det\left(\begin{bmatrix}
\frac{-b(c+f)}{d} & 0 \\
\frac{d(a-e)}{b} & 0
\end{bmatrix} - \lambda \begin{bmatrix}
a & 0 \\
0 & b
\end{bmatrix}\right) = \lambda^2 + (c+f)(a-e) = 0,
\]

which implies that 

\(J_1\) has eigenvalues \(\lambda_1 = \sqrt{(c+f)(a-e)}\) and \(\lambda_2 = -\sqrt{(c+f)(a-e)}\).

In the numerical simulation section we will use the eigenvalues of 

\(J_0\) and 

\(J_1\) to determine the local stability of the equilibrium points \((0,0)\) and 

\(\left(\frac{c+f}{d}, \frac{a-e}{b}\right)\).
PARAMETER ESTIMATION

In this section we will estimate the values of the parameters $a$ and $b$, particularly, for the two spotted spider mite, and the values of the parameters $c$ and $d$ for the fallacis mite.

First we researched the two spotted spider mite, and its intrinsic increase rate or $a$. On (2) we found the information that the initial population of the two spotted spider mite will increase by 35 fold to 71 fold in 21 days, under optimal conditions. We used the time period of 46 days. For the calculations, we used the lower bound of 35 fold. We assumed that our initial population is 10 two spotted spider mites (or it can be any initial number $n$), after 21 days there will be 350 spider mites. This is under the assumption that they hatch and die at the same time. We took 350 (population after 21 days) minus 10 (the initial population) and divided that result by 10 (the initial population), we got 3400% or 34. Then we took the time period of 46 days and divided it by 21 (time it took to reach our total population), we got 2.19. Finally, multiply 2.19 by 3400% to get intrinsic increase rate of 7446% or 74.46.

Second, we researched the fallacis mite and the increase rate of its population due to predation or $d$. On (1) we found the information that 10 fallacis mites will turn into 200-500 mites in 14 days, under optimal conditions. Again, the time period is 46 days and we used the lower bound of 200 mites. We took the 200 (population after 14 days) minus 10 (the initial population) and divided that result by 10, we got 1900% or 19. Then take the time period of 46 days and divide it by the time it takes for the 10 mites to become 200 mites, 14 days to get 3.29. Finally we multiplied 3.29 by 1900%, to get the increase rate due to predation of 6251% or 62.51.

Third, we examined the intrinsic decrease rate of the fallacis mite or $c$. There are two ways of calculating this number. First, it can be done using the following formula; (total population-original) divided by total population, then subtract that number from 1. The second way, and the one we used, is the initial population divided by total population. For this model, our initial population was 635, to obtain the total population, we multiplied the initial population by the growth rate (635 times 6251%). Our total population is 39642. Finally, we divided the 635 by 39642, to obtain the intrinsic decrease rate of 1.6% or .016.

Finally, we examined the decrease rate of the two spotted spider mite population due to predation or $b$. In (2) we found the information about the eating habits of fallacis mites. As a juvenile, a fallacis mite will eat 4 two spotted spider mites per day and as an adult, they will eat 15 two spotted spider mites per day. On average we calculated one fallacis mite will eat 602 two spotted spider mites in a lifetime. Again we used the initial population of 635 fallacis mites. We multiplied 602 by 635, 382270, which is the total of two spotted spider mites that 635 fallacis mites will eat in a lifetime. To obtain the consumption rate we took the total consumption of prey divided by population of spider mite. To get the total populations of the spider mite for this model, we started off with 100 spider mites. After a time period of 46
days, there will be 7446 spider mites. Finally, we divided 382270 by 7446 which yields the decrease rate due to predation of 5130% or 51.3.

Now, we have the values for our parameters \( a = 74.46, \ b = 51.3, \ c = 0.016 \) and \( d = 62.51 \).

**NUMERICAL SIMULATION RESULTS**

In this section we will do numerical simulation for various values of the free parameters \( e \) and \( f \). First, we consider when the agricultural field is not treated by pesticides:

**(i) \( e = 0 \) and \( f = 0 \)**

The prey (the two spotted spider mite) is controlled by the predator (the fallacis mite) only. In this case, the trivial equilibrium point \((0,0)\) is a saddle, because \( \lambda_1 = a - e = 74.46 > 0 \) and \( \lambda_2 = c - f = 0.016 < 0 \). And the positive equilibrium point \( \left( \frac{c + f}{d}, \frac{a - e}{b} \right) = (0.000259, 1.4514) \) is a center because \( \lambda_1 = \sqrt{(c + f)(e - a)} = \sqrt{1.191} \) and \( \lambda_2 = -\sqrt{(c + f)(e - a)} = -\sqrt{1.191} \).

Figure 1 (a) is a phase portrait. For initial values close to the positive equilibrium point, the trajectories are simple closed curves enclosing the equilibrium point. If the initial point is far from the equilibrium point, the trajectory eventually will converge to a stable limit cycle (or periodic) centered at the positive equilibrium point. For example in Figure 1 (c) it converges to periodic trajectory after a time \( t = 16 \). From Figure 1 (b) and Figure 1 (c) it can be seen that the predator population is much higher than the prey population, both for larger and smaller initial populations. If the initial population is higher (for both prey and predator) the predation successfully can control (keep it at lower population level) the pest population. Next we will investigate how the pesticide will affect both prey and predator.
Figure 2. Phase portrait and population versus time graphs when $e=40$ and $f=20$.

(iii) $e=60$ and $f=60$

For $e=60$ and $f=60$, the trivial equilibrium, $(0,0)$, is saddle, and the positive equilibrium point $(\frac{c+f}{d}, \frac{a-e}{b})=(0.9601, 0.2818)$ is a center.

Figure 3 (c) suggests that there is a seasonal (periodic) outbreak of the prey (its population is higher than the predator population). For the same initial population size this was not observed when pesticide was not used ($e=0$ and $f=0$). For initial population close to the positive equilibrium the trajectories are simple closed curves, at each time $t$ the prey population is higher than the predator population (Figure 3 (b)). The results are interesting, but should not be a surprise, because the increase in $f$ causes decreases in predation which can cause an increase in the prey population.
Figure 3. Phase portrait and population versus time when $e=60$ and $f=60$.

**iv** $e=40$ and $f=60$

For $e=40$ and $f=60$, the trivial equilibrium, $(0,0)$, is a saddle, and the positive equilibrium point $(\frac{c+f}{d}, \frac{a-e}{b})=(0.9601, 0.6717)$ is a center.

Compare to Figure 3, in Figure 4 the predator population size is higher (both in (b) and (c)). This is expected because we lowered $e$ to 40 and more prey survived (more food for predator) and that increased the predator population.
Figure 4. Phase portrait and population versus time graphs when e=40 and f=60.

(v) e=25 and f=55
For e=25 and f=55, the trivial equilibrium, (0,0), is a saddle, and the positive equilibrium point \( \left( \frac{c+f}{d}, \frac{a-e}{b} \right) = (0.8801, 0.9647) \) is a center.

The value of e is lowered to 25 and f is lowered to 55. The population dynamics of the prey and predator in Figure 5 is similar to Figure 4 but the prey population is lower. This is expected because as f increases predation will increase (due increase in predator population), which will cause prey population to be decreased.
Figure 5. Phase portrait and population versus time when $e=25$ and $f=55$.

(vi) $e=80$ and $f=25$

For $e=80$ and $f=25$, $e$ is greater than $a$ means the pesticide killed all the prey population. The trivial equilibrium point $(0,0)$ is a unique equilibrium point and it is a sink (attractor), because $\lambda_1 = -c-f = -5.54 < 0$ and $\lambda_2 = -c-f = 25.016 < 0$. As shown in Figure 6, for any initial population size, both the prey and the predator will become extinct (eventually enter the $(0,0)$ equilibrium state).
CONCLUSION

In the absence of the predator (fallacis mite) the prey (two spotted spider mite) population will grow exponentially (explode) unless it is treated by the pesticide with $e>a$. If $e>a$, still there will be a pest (prey) outbreak even if it is treated with the pesticide.

In the presence of the predator, Figure 1 shows, even if there is no pesticide treatment the prey population is kept at low population size because of the predation. If there is a pesticide treatment with $e=40$ and $f=50$, as in Figure 2, the predator population decreased but the prey population increased. This is because the decrease in predation due the death of the predators by the pesticide.

In Figure 3 and Figure 4, $f$ is increased to 60, the death rate of the predator will be higher (and consequently lower predation). Because of this the prey population co-exists at higher population. For lower $f$ values (as in Figure 5), the death rate of the predator will decrease (and consequently predation will increase) and that will keep the prey population at low level, but not significantly lower than the size of prey population when pesticide is not used.

Therefore, in the presence of the predator, the predation alone can help to keep the prey population at a low level. In some cases using pesticide

Figure 6. Phase portrait and population versus time graphs when $e=80$ and $f=25$. 

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can promote seasonal (periodic) pest (prey) outbreak. If the pesticide is used extensively enough so that $c$ is larger than $a$ (as in Figure 6) both the prey and predator will be eliminated, which may not be desirable. The extinction of the predator may cause another pest (prey) outbreak in the surrounding environment. The results of our simplified model suggest that many cases have to be taken into consideration when pesticides are used in agricultural environment.

REFERENCES