Georgia Journal of Science

Volume 81 *Scholarly Contributions from the Membership and Others*

Article 3

2023

A Mathematical Model for Mosquito Population Dynamics

Christian Evans
Valdosta State University

Jemal Mohammed-Awel
Morgan State University, Baltimore

Andreas Lazari
Valdosta State University, alazari@valdosta.edu

A Mathematical Model for Mosquito Population Dynamics

Acknowledgements

Jemal Mohammed-Awel acknowledges the support, in part, of the National Science Foundation (NSF) (DMS-2052355).

A MATHEMATICAL MODEL FOR MOSQUITO POPULATION DYNAMICS

Christian Evans and Andreas Lazari*
Department of Mathematics, Valdosta State University, Valdosta, Georgia 31698
Corresponding author: alazari@valdosta.edu

Jemal Mohammed-Awel
Department of Mathematics, Morgan State University, Baltimore, Maryland 21251

ABSTRACT

In this study, a deterministic mathematical model for mosquito population dynamics is presented. The use of chemical insecticide to control population is incorporated into the model. It is assumed that there is insecticide sensitive (sensitive-type) and insecticide resistant (resistant-type) mosquitoes in the environment. Conditions for the existence and stability of four equilibria of the model have been established. Numerical simulations are carried out to confirm the analytical results and the implications, in terms of mosquito control in the environment, are discussed.

INTRODUCTION

With nearly three thousand species across every part of the world, except the Antarctic, mosquito population numbers peak over all other animals except for ants and termites (Gates 2014). Out of more than 200 species of mosquitos in the US, only 12 can transmit diseases (Centers for Disease Control and Prevention 2020). By the ability to fly, mosquitos can spread mosquito-borne disease parasites easier than other person-to-person transmitted diseases such as HIV and Ebola. Mosquitos bite at all hours indoors and outdoors (Mohammed-Awel et al. 2018).

Female adult *Anopheline* mosquitoes transmit malaria parasitic from infected humans to susceptible humans (World Health Organization 2022). After they mate, female adult mosquitoes bite humans (and other animals) and take a blood meal to obtain the protein necessary for egg development and egg laying. Mosquitoes acquire the parasite (*Plasmodium*) infection by taking blood meals from an infected human. After the parasites complete the development period within the mosquito, the mosquito can subsequently pass the disease to a susceptible human (Environmental Protection Agency 2021). In 2021, malaria causes an estimated 247 million cases globally, and results in 619,000 deaths (World Health Organization 2022). In 2020, 95% of malaria cases and 96% malaria deaths occurred in African region (World Health Organization 2022). Moreover, an estimated 80% of the deaths occurred in children under the age of 5 years in this region (World Health Organization 2022). Mosquitoes also transmit other diseases (such as West Nile, Zika, dengue or yellow fever, and encephalitis) (Lee et al. 2018). These mosquito-borne diseases cost over half a million human lives every year (Lee et al. 2018; Price 2019). Furthermore, these diseases cost billions of dollars every year due to health-related productivity (Lee et al. 2018; Price 2019).

Mosquito control methods, such as Indoor residual insecticide spray (IRS with DDT), insecticide-treated bed-nets (ITNs) and long-lasting insecticide-treated bed-nets (LLINs) are among the strategies used to control mosquito-borne diseases (Mohammed-Awel and Numfor 2017; Mohammed-Awel et al. 2018; Mohammed-Awel and Gumel

2019). For example, malaria disease burden was reduced dramatically between 2000 and 2015, and the reduction was mainly due to the use of ITNs and LLINs (Mohammed-Awel and Gumel 2019). Furthermore, malaria is also treated with drugs to kill the malaria parasite. The most common prescription drugs used to treat malaria are Chloroquine and Artemisinin-based combination therapies (ACTs). However, the emergence of drug resistance in disease parasites and insecticide resistance in mosquitoes pose a threat to the effectiveness of drug treatment and LLINs/IRS to control malaria (Mohammed-Awel and Numfor 2017; Mohammed-Awel et al. 2018; Mohammed-Awel and Gumel 2019).

The Rose model was the first mathematical model that was developed to study malaria transmission and control (Ross 1916). There are several mathematical models in the literature (mainly extension of Rose's model) that have been developed to study malaria transmission and control by incorporating human or mosquito population (or both) (Ross and Robert 1916; Macdonald 1957; Morgan and Robert 2015; Mohammed-Awel et al. 2018; Mohammed-Awel and Gumel 2019). Furthermore, a few mathematical models have been developed to study the impact of insecticide resistance in mosquito and malaria control (Mohammed-Awel et al. 2018; Mohammed-Awel and Gumel 2019).

In this study, we developed a mathematical model for adult mosquito population dynamics that incorporates the population of sensitive-type and resistant-type mosquitoes to insecticides. It is assumed that sensitive-type mosquitoes can mutate and became resistant-type at a constant rate, and the use of chemical adulticide is incorporated into the model. Basic model properties, derivation of the model equilibria, and numerical simulations are carried out. Using numerical simulations, scenarios for the spread and management of insecticide resistance in mosquito population are discussed.

MODEL DERIVATION

The model to be developed is for the spread of insecticide resistance in mosquitoes where chemical insecticide (IRS) is used to control mosquito population growth.

In this study, it is assumed that mosquitos' genotype for insecticide resistance is determined by the presence or absence of a resistant (R) allele and a sensitive (S) allele. Mosquitoes are said to be resistant to insecticide if, upon contact, the insecticide do not kill or the ability of the insecticide to kill them is greatly reduced or eliminated. The total adult mosquito population at time t, denoted by $M(t)$, and mosquitoes are classified as insecticide sensitive mosquitoes ($M_S(t)$) and insecticide resistant mosquitoes ($M_R(t)$), so that the total mosquito population at time t is $M(t) = M_S(t) + M_R(t)$.

It is assumed that sensitive-type mosquitoes can mutate and became resistant-type at a rate m. Furthermore, it is assumed that resistance is inherited (that is, a resistant female adult mosquito vertically produces resistant offspring [7]. For both sensitive-type and resistant-type mosquitoes, the following logistic growth (birth) functions, $B_S(t)$ and $B_R(t)$, are chosen.

$B_S(t) = b_S\left(1 - \frac{M}{K_M}\right)$ and $B_R(t) = b_R\left(1 - \frac{M}{K_M}\right)$

where $b_S > 0$ and $b_R > 0$ are the production (birth) rates of new adult insecticide sensitive-type and insecticide resistant-type mosquitoes, respectively. The parameter $K_M > 0$ is the environmental carrying capacity of adult mosquitoes.

Both sensitive-type and resistant-type mosquitoes suffer natural death at a rate μ_S and μ_R, respectively. Furthermore, it is assumed that insecticide sensitive-type mosquitoes suffer insecticide induced death at a rate $u_i\delta_{iS}$, where u_i is a proportion of houses (indoors) sprayed with IRS in the community, and δ_{iS} death rate of insecticide sensitive-type mosquitoes due to the use of IRS. Similarly, due to the use of IRS, insecticide resistant-type mosquitoes suffer additional mortality at a rate of rate $u_i\delta_{iR}$ where δ_{iR} is death rate of insecticide resistant-type mosquitoes due to the use of IRS.

The model for the mosquito population dynamics, in the presence of IRS, is given by the following deterministic system of non-linear differential equations:

$$\frac{dM_S}{dt} = b_S\left(1 - \frac{M}{K_M}\right)M_S - mM_S - (u_i\delta_{iS} + \mu_S)M_S$$

$$\frac{dM_R}{dt} = b_R\left(1 - \frac{M}{K_M}\right)M_R + mM_S - (u_i\delta_{iR} + \mu_R)M_R \qquad (1)$$

A schematic diagram of the model is depicted in Figure 1 (and the state variables and parameters of the model are described in Tables I and II, respectively).

Table I. State variables.

State Variable	Description
$M_S(t)$	Population of sensitive-type mosquitoes
$M_R(t)$	Population of resistant-type mosquitoes
$M(t) = M_S(t) + M_R(t)$	Total mosquito population

Table II. Parameters of the model.

Parameter	Description	Range of values	Baseline value	Source
b_S	Birth rate (*per day*) of sensitive-type mosquito	varies	0.22, 4.22	Assumed
b_R	Birth rate (*per day*) of resistant-type mosquito	varies	0.2, 2.22, 3.22	Assumed
u_i	Proportion of insecticide usage in the environment ($0 \le u_i \le 1$)	$0 \le u_i \le 1$	0.9	Estimated
δ_{iS}	Death rate of sensitive-type mosquitoes due to the use of insecticide	varies	0.04, 0.07, 0.8	Assumed
δ_{iR}	Death rate (*per day*) of resistant-type mosquitoes due to the use of insecticide	varies	0.4, 0.11	Assumed
m	The rate (*per day*) at which insecticide sensitive allele (S) mutate to insecticide resistant allele (R)	$0 \le m \le 0.005$	0 and 0.002 *per day*	Estimated
μ_S	Natural death rate (*per day*) of sensitive-type mosquito	$\frac{1}{21} \le \mu_S \le \frac{1}{14}$	1/14 *per day*	Estimated
μ_R	Natural death rate (*per day*) of resistant-type mosquito	$\frac{1}{14} \le \mu_S \le \frac{1}{7}$	1/7, 1/14 *per day*	Estimated
K_M	Environmental carrying capacity of mosquitoes	varies	10^5	Estimated

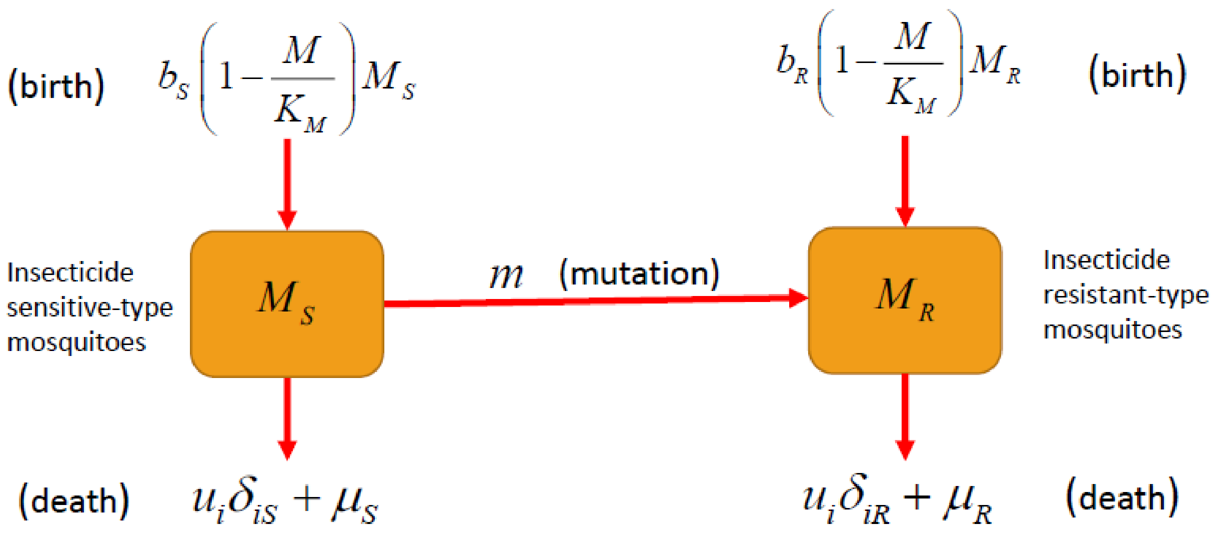


Figure 1: Model schematic diagram.

MODEL ANALYSIS

The model (1) for mosquito populations and all its associated parameters are non-negative.

Theorem 4.1**:** Solutions of the model system (1) with non-negative initial data are non-negative for all time t > 0 (Mohammed-Awel et al. 2018).

Equilibrium solution: an equilibrium solution is a constant solution of (1) that satisfies equation (2). That is, at equilibrium solution, both variables M_S and M_R are constant.

$$b_S\left(1-\frac{M}{K_M}\right)M_S - mM_S - (u_i\delta_{iS}+\mu_S)M_S = 0,$$
$$b_R\left(1-\frac{M}{K_M}\right)M_R + mM_S - (u_i\delta_{iR}+\mu_R)M_R = 0. \qquad (2)$$

Equilibria*:* the equilibria of the model equation (1) are obtained by setting the right side of equations (1) equal to zero and solving the equation, that is, by solving equation (2). Conditions for the existence of four different equilibria are derived below.

Trivial equilibrium*:* it can be verified that $M_S = M_R = 0$ is a solution of the model equation (1) and it is an equilibrium, we call this trivial equilibrium, denoted by E_0 where, $E_0 = (0,0)$.

Obviously, E_0 satisfies equation (2).

Sensitive-type-only equilibrium: when there is no mutation (that is, $m=0$). We can derive an insecticide sensitive only equilibrium by setting $M_R = 0$ in equation (2) and solve for M_S.

If $M_R = 0$ and $m=0$, equation (2) is reduced to:

$$b_S\left(1-\frac{M_S}{K_M}\right)M_S-(u_i\delta_{iS}+\mu_S)M_S=0.$$

By factoring M_S, we obtain,

$$\left[b_S-\frac{b_SM_S}{K_M}-(u_i\delta_{iS}+\mu_S)\right]M_S=0,$$

which is equivalent to

$$\left[M_S-K_M+\frac{K_M(u_i\delta_{iS}+\mu_S)}{b_S}\right]\left(\frac{-b_SM_S}{K_M}\right)=0.$$

Combine the terms $-K_M$ and $\frac{K_M(u_i\delta_{iS}+\mu_S)}{b_S}$, and simplify gives

$$\left[M_S-\frac{K_M(u_i\delta_{iS}+\mu_S)}{b_S}\left(\frac{b_S}{u_i\delta_{iS}+\mu_S}-1\right)\right]\left(\frac{-b_SM_S}{K_M}\right)=0,$$

If we define $\Re_M^S=\frac{b_S}{u_i\delta_{iS}+\mu_S}$, then the above equation becomes

$$\left[M_S-\frac{K_M}{\Re_M^S}(\Re_M^S-1)\right]\left(\frac{-b_SM_S}{K_M}\right)=0\,. \qquad (3)$$

Therefore, $M_S=\frac{K_M}{\Re_M^S}\left(\Re_M^S-1\right)$ or $M_S=0$ are solutions of (3). The solution $M_S=\frac{K_M}{\Re_M^S}\left(\Re_M^S-1\right)$ is positive if and only if $\Re_M^S>1$. Therefore, the sensitive-type-only equilibrium exists if $\Re_M^S>1$, and it is denoted by $E_S=(M_S^*,0)=\left(\frac{K_M}{\Re_M^S}(\Re_M^S-1),0\right)$.

Resistant-only equilibrium: the resistant only equilibrium is obtained by setting $M_S=0$ in equation (2) and solve for M_R to obtain $\left(0,M_R^*\right)$

If $M_S=0$, equation (2) is reduced to:

$$b_R\left(1-\frac{M_R}{K_M}\right)M_R-(u_i\delta_{iR}+\mu_R)M_R=0.$$

By factoring M_S, we obtain

$$\left[b_R-\frac{b_RM_R}{K_M}-(u_i\delta_{iR}+\mu_R)\right]M_R=0,$$

which is equivalent to

$$\left[M_R - K_M + \frac{K_M(u_i\delta_{iR}+\mu_R)}{b_R}\right]\left(\frac{-b_R M_R}{K_M}\right) = 0.$$

Combine the terms $-K_M$ and $\frac{K_M(u_i\delta_{iR}+\mu_R)}{b_R}$, and then simplify, gives

$$\left[M_R - \frac{K_M(u_i\delta_{iR}+\mu_R)}{b_R}\left(\frac{b_R}{u_i\delta_{iR}+\mu_R} - 1\right)\right]\left(\frac{-b_R M_R}{K_M}\right) = 0.$$

If we define $\Re_M^R = \frac{b_R}{u_i\delta_{iR}+\mu_R}$, then we have

$$\left[M_R - \frac{K_M}{\Re_M^R}(\Re_M^R - 1)\right]\left(\frac{-b_R M_R}{K_M}\right) = 0. \qquad (4)$$

Therefore, the solution of equation (4) is $M_R = \frac{K_M}{\Re_M^R}\left(\Re_M^R - 1\right)$ or $M_R = 0$. It is clear that $M_R = \frac{K_M}{\Re_M^R}\left(\Re_M^R - 1\right)$ is positive is positive if $\Re_M^R > 1$. Therefore, the sensitive-type-only equilibrium exists if and only if $\Re_M^R > 1$, and it is $E_R = (0, M_R^*) = \left(0, \frac{K_M}{\Re_M^R}(\Re_M^R - 1)\right)$.

<u>Co-existence equilibrium</u>: at the co-existence equilibrium where both sensitive-only and resistant-only mosquitoes exist, both M_S and M_R are positive ($M_S > 0$ and $M_R > 0$) and can be obtained by solving equation (2) for M_S and M_R.

$$b_S\left(1 - \frac{M}{K_M}\right)M_S - mM_S - (u_i\delta_{iS} + \mu_S)M_S = 0, \qquad (5)$$
$$b_R\left(1 - \frac{M}{K_M}\right)M_R + mM_S - (u_i\delta_{iR} + \mu_R)M_R = 0\,. \quad (6)$$

From (5), we have $[b_S\left(1 - \frac{M}{K_M}\right) - (m + (u_i\delta_{iS} + \mu_S))]M_S = 0$,

if $M_S \neq 0$, the equation equivalent to $\left(1 - \frac{M}{K_M}\right) = \frac{m+u_i\delta_{iS}+\mu_S}{b_S}$. $\qquad$ (7)

By solving the above equation for M we obtain $M = K_M - \frac{K_M(m+u_i\delta_{iS}+\mu_S)}{b_S}$. This can be simplified to $M = \frac{K_M(m+u_i\delta_{iS}+\mu_S)}{b_S}\left[\frac{b_S}{m+u_i\delta_{iS}+\mu_S} - 1\right]$.

If we define $\Re_M^{Sm} = \frac{b_S}{m+u_i\delta_{iS}+\mu_S}$, then we have $M = \frac{K_M}{\Re_M^{Sm}}[\Re_M^{Sm} - 1]$. $\qquad$ (8)

Observe that $M > 0$ if and only if $\Re_M^{Sm} > 1$. Now, using (7) replace $\left(1 - \frac{M}{K_M}\right)$ in (6) by $\frac{m+u_i\delta_{iS}+\mu_S}{b_S}$ gives, $\quad b_R\frac{m+u_i\delta_{iS}+\mu_S}{b_S}M_R + mM_S - (u_i\delta_{iR} + \mu_R)M_R = 0.$

Use the above equation to solve M_S in terms M_R and obtain.

$$M_S = \frac{b_R}{m}\left[\frac{(u_i\delta_{iR}+\mu_R)}{b_R} - \frac{(m+u_i\delta_{iS}+\mu_S)}{b_S}\right]M_R.$$

Using the nations $\mathfrak{R}_M^R$ and $\mathfrak{R}_M^{Sm}$, we can re-write the above equation as

$$M_S = \frac{b_R}{m}\left[\frac{1}{\mathfrak{R}_M^R} - \frac{1}{\mathfrak{R}_M^{Sm}}\right]M_R,$$

which is the same as

$$M_S = \frac{b_R}{m\mathfrak{R}_M^R\mathfrak{R}_M^{Sm}}[\mathfrak{R}_M^{Sm} - \mathfrak{R}_M^R]M_R\,. \qquad (9)$$

From (8), we have

$$M_S + M_R = \frac{K_M}{\mathfrak{R}_M^{Sm}}[\mathfrak{R}_M^{Sm} - 1]. \qquad (10)$$

Then, from equations (9) and (10), we obtain

$$\frac{b_R}{m\mathfrak{R}_M^R\mathfrak{R}_M^{Sm}}[\mathfrak{R}_M^{Sm} - \mathfrak{R}_M^R]M_R + M_R = \frac{K_M}{\mathfrak{R}_M^{Sm}}[\mathfrak{R}_M^{Sm} - 1]. \qquad (11)$$

Solve equation (11) for M_R and simplify to obtain,

$$M_R = \frac{m\mathfrak{R}_M^R K_M[\mathfrak{R}_M^{Sm}-1]}{m\mathfrak{R}_M^R\mathfrak{R}_M^{Sm}+b_R[\mathfrak{R}_M^{Sm}-\mathfrak{R}_M^R]}. \qquad (12)$$

Finally, using equations (10) and (12), M_S can be solved and

$$M_S = \frac{K_M b_R[\mathfrak{R}_M^{Sm}-\mathfrak{R}_M^R][\mathfrak{R}_M^{Sm}-1]}{\mathfrak{R}_M^{Sm}[m\mathfrak{R}_M^R\mathfrak{R}_M^{Sm}+b_R(\mathfrak{R}_M^{Sm}-\mathfrak{R}_M^R)]}. \qquad (13)$$

Therefore, from (12) and (13), we conclude that the co-existence equilibrium

$$E_C = (M_S^*, M_R^*) = \left(\frac{K_M b_R[\mathfrak{R}_M^{Sm} - \mathfrak{R}_M^R][\mathfrak{R}_M^{Sm} - 1]}{\mathfrak{R}_M^{Sm}[m\mathfrak{R}_M^R\mathfrak{R}_M^{Sm} + b_R(\mathfrak{R}_M^{Sm} - \mathfrak{R}_M^R)]}, \frac{m\mathfrak{R}_M^R K_M[\mathfrak{R}_M^{Sm} - 1]}{m\mathfrak{R}_M^R\mathfrak{R}_M^{Sm} + b_R[\mathfrak{R}_M^{Sm} - \mathfrak{R}_M^R]}\right)$$

exists if an only if $\mathfrak{R}_M^{Sm} > 1$ and $\mathfrak{R}_M^{Sm} > \mathfrak{R}_M^R$.

All the above discussions are summarized in Theorem 4.2 below.

Theorem 4.2: Consider the model (1).

1. The sensitive only equilibrium E_S exists if and only if there is no mutation (i.e., $m = 0$) and $\mathfrak{R}_M^S > 1$.
2. The resistant only equilibrium E_R exists if and only if $\mathfrak{R}_M^R > 1$.
3. The co-existence equilibrium E_C exists if and only if $m > 0$, $\mathfrak{R}_M^{Sm} > 1$ and $\mathfrak{R}_M^{Sm} > \mathfrak{R}_M^R$.

STABILTY OF THE EQUILIBRIA

The local stability of the model (1) at the equilibria E_0, E_S, E_R and E_C can be established by linearizing the non-linear system (1) at each equilibrium. The stability of each equilibrium is determined based on the eigenvalues of the Jacobian matrix of the linearized system (Morgan 2015).

Theorem 5.1: Consider the model (1).

1. The trivial equilibrium, E_0, is stable if $\Re_M^{Sm} < 1$ and $\Re_M^R < 1$, and unstable if $\Re_M^{Sm} > 1$ or $\Re_M^R > 1$.
2. Let $\Re_M^S > 1$ and $m = 0$, then the sensitive only equilibrium E_S (which exists) is stable if $\Re_M^S > \Re_M^R$ and is unstable if $\Re_M^S < \Re_M^R$.
3. Let $\Re_M^R > 1$ and $m > 0$, then the resistant only equilibrium E_R (which exists) is stable if $\Re_M^R > \Re_M^{Sm}$ and is unstable if $\Re_M^R < \Re_M^{Sm}$.

Proof:

1. For the trivial equilibrium $E_0 = (0,0)$, the Jacobian matrix (or the coefficient matrix of the linearized system at E_0) is (Morgan 2015) :

$$\begin{pmatrix} (K_1 + m)(\Re_M^{Sm} - 1) & 0 \\ m & K_2(\Re_M^R - 1) \end{pmatrix},$$

where $K_1 = u_i\delta_{iS} + \mu_S$ and $K_2 = u_i\delta_{iR} + \mu_R$. The eigenvalues of the Jacobian are $\lambda_1 = (K_1 + m)(\Re_M^{Sm} - 1)$ and $\lambda_2 = K_2(\Re_M^R - 1)$. Both eigenvalues λ_1 and λ_2 are negative if and only if $\Re_M^{Sm} < 1$ and $\Re_M^R < 1$, respectively. Therefore, the trivial equilibrium $E_0 = (0,0)$ is stable if $\Re_M^{Sm} < 1$ and $\Re_M^R < 1$ and unstable if $\Re_M^{Sm} > 1$ or $\Re_M^R > 1$.

2. For the sensitive only equilibrium E_S, the Jacobian matrix is:

$$\begin{pmatrix} -K_1(\Re_M^S - 1) & -K_1(\Re_M^S - 1) \\ 0 & -\frac{K_2}{\Re_M^S}(\Re_M^S - \Re_M^R) \end{pmatrix}.$$

The eigenvalues are $\lambda_1 = K_1(1 - \Re_M^S)$ and $\lambda_2 = \frac{K_2}{\Re_M^S}(\Re_M^R - \Re_M^S)$. Since $\Re_M^S > 1$, $\lambda_1 < 0$, the second eigenvalue λ_2 is negative if $\Re_M^S > \Re_M^R$. Consequently, the sensitive only equilibrium E_S is stable if $\Re_M^S > \Re_M^R$ and unstable if $\Re_M^S < \Re_M^R$.

3. Similarly, for the resistant only equilibrium E_R, the Jacobian matrix is:

$$\begin{pmatrix} -\frac{(K_1+m)}{\Re_M^R}(\Re_M^R - \Re_M^{Sm}) & 0 \\ -\frac{b_r}{\Re_M^R}(\Re_M^R - 1) + m & -K_2(\Re_M^R - 1) \end{pmatrix}.$$

The eigenvalues are $\lambda_1 = \frac{(K_1+m)}{\Re_M^R}(\Re_M^{Sm} - \Re_M^R)$ and $\lambda_2 = K_1(1-\Re_M^R)$. Since $\Re_M^R > 1$, $\lambda_2 < 0$, and the first eigenvalue λ_1 is negative if $\Re_M^R > \Re_M^{Sm}$. Therefore, the resistant only equilibrium is stable if $\Re_M^R > \Re_M^{Sm}$ and is unstable if $\Re_M^R < \Re_M^{Sm}$.

The stability of the co-existence equilibrium E_C is not show in this study since the algebraic expression involved in the calculations are complicated. However, we present numerical simulations in Section 5 (Figures 5 (a) & (b)). Through numerical simulations in Figure 5, we show that if $m > 0$, $\Re_M^{Sm} > 1$ and $\Re_M^{Sm} > \Re_M^R$, numerical solutions converge to the coexistence equilibrium E_C for numerous arbitrary chosen initial conditions for the sensitive-type and resistant-type mosquitoes.

NUMERICAL SOLUTIONS

In this section, for the given (chosen) parameter values and initial population values, the following scenarios are simulated.

I. **Case 1**: A control strategy that causes mosquito extinction from the environment,

II. **Case 2**: Competitive exclusion in absence of mutation: a phenomenon where sensitive-only mosquito drives resistant-only to extinction.

III. **Case 3**: Competitive exclusion in absence of mutation: a phenomenon where resistant-only mosquito drives sensitive-only to extinction.

IV. **Case 4: C**o-existence of the sensitive and resistant mosquitoes.

CASE 1 SCENARIO

Simulation results in Figure 2 show that mosquitoes go to extinction at equilibrium for any initial condition (M_S, M_R) where $M_S \geq 0$ and $M_R \geq 0$. For this parameter group, the values of the thresholds $\Re_M^S$, $\Re_M^R$, and $\Re_M^{Sm}$ are 0.28, 0.46, and 0.28, respectively. Since the value of each threshold less than unity, then by Theorem 4.2 the equilibria E_S, E_R and E_C do not exist. Also, since the conditions $\Re_M^{Sm} < 1$ and $\Re_M^R < 1$ in Theorem 5.1 are satisfied, the trivial equilibrium E_0 is stable. The simulation result in figure 2 (a) show that the solution trajectories for several initial values converge to the trivial equilibrium E_0 in the M_S-M_R phase plane. Moreover, in Figure 2 (b), we run simulations for 100 randomly chosen initial values of M_S and M_R for 50 days. The result shows that eventually both M_S and M_R converges to zero for all initial values (Figure 2 (b)). This result agrees with the results in Figure 2 (a). Thus, the numerical simulation confirms that the trivial equilibrium is stable equilibrium if $\Re_M^{Sm} < 1$ and $\Re_M^R < 1$. The biological implication is that, in this scenario, the chemical insecticide-based mosquito control strategy eliminates the mosquito population from the environment.

CASE 2 SCENARIO

For the parameter values in Figure 3, the values of the thresholds $\Re_M^S$, $\Re_M^R$, and $\Re_M^{Sm}$ are 24.76, 16.51 and 24.76, respectively. Since $\Re_M^S > 1$, $\Re_M^R > 1$ and $m = 0$, by Theorem 4.2, the trivial (E_0), sensitive-only (E_S) and resistant-only (E_R) equilibria exist. However, the co-existence equilibrium (E_C) does not exist in this scenario. By Theorem 5.1, E_0 and E_R are unstable (since $1 < \Re_M^R < \Re_M^{Sm}$), and E_S stable (since $\Re_M^{Sm} > \Re_M^R$). This theoretical result is also confirmed numerically in Figures 3 (a) & (b). In Figure 3 (a), the direction field and the solution curves, for several initial values, converge to the sensitive-only equilibrium E_S (in the M_S-axis) in the M_S-M_R plane. Furthermore, in the direction field plane in Figure 3 (a), if the initial value is in the M_R-axis, the simulation converges to the resistant-only equilibrium. To confirm the result observed in Figure 3 (a), we run simulations for 100 randomly chosen initial values of M_S and M_R for 200 days. The result, in Figure 3 (b), shows that the simulation converges to the same sensitive-only equilibrium E_S as in Figure 3 (a). This result confirms that the sensitive-only equilibrium, E_S, is stable equilibrium, whereas the trivial (E_0) and resistant-only (E_R) equilibria are unstable. The biological implication in this scenario is that the chemical insecticide-based control strategy does not cause the problem of insecticide resistance in mosquitoes in the environment, however, the strategy does not control the mosquito population (or do not reduce mosquito abundance) in the environment. The interpretation of this scenario is that if both resistant and sensitive mosquitoes are present initially, the control strategy causes a phenomena that sensitive-only mosquito drives resistant-only to extinction in the environment (observe that $\Re_M^S > \Re_M^R$).

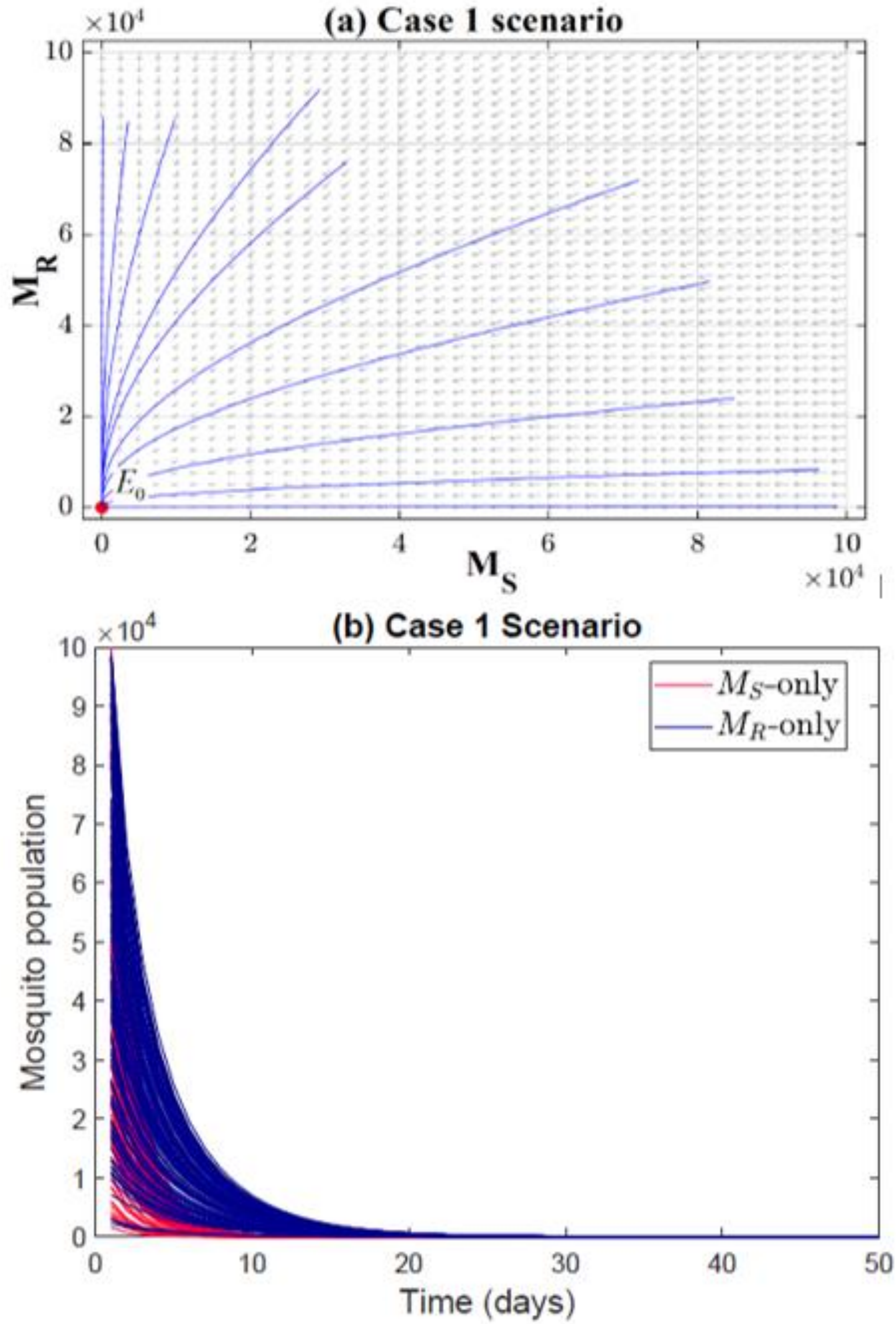
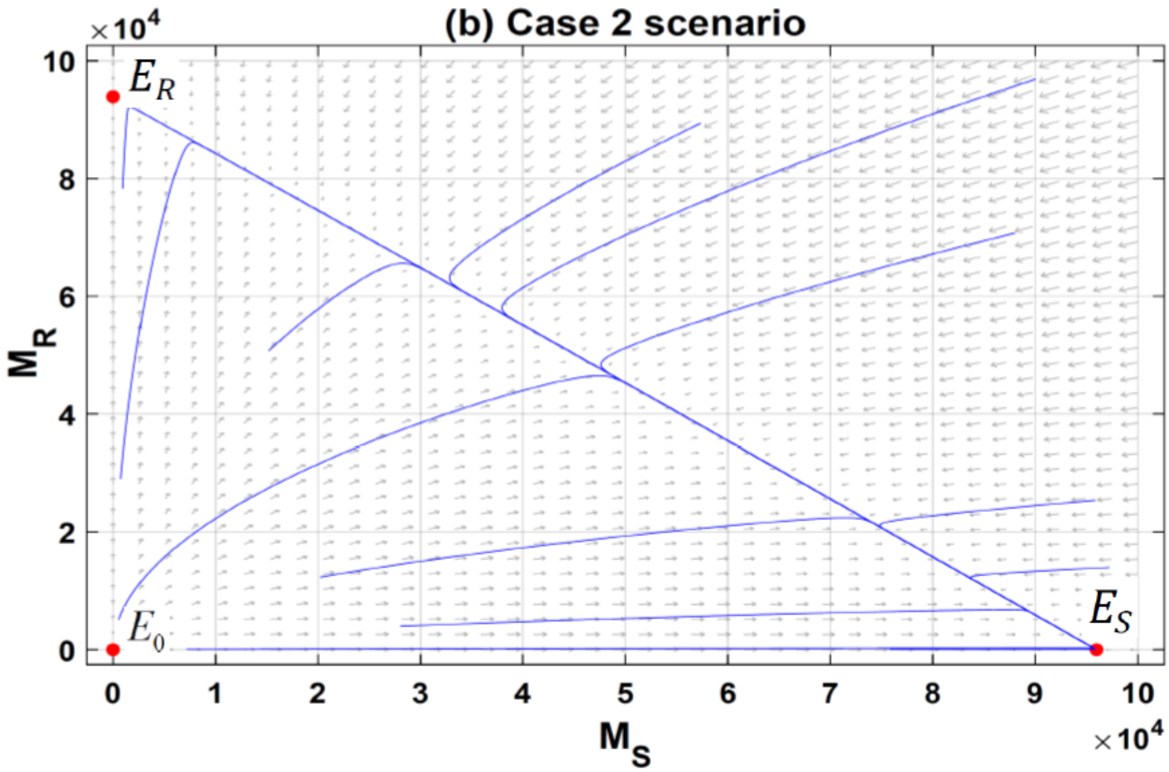


***Figure 2**:* Numerical simulation of the model system (1) for **Case 1 scenario**.

Figure 2 shows numerical simulation of the model system (1) for **Case 1 scenario:** (a) direction field and solution curves and (b) solution curves versus time for 100 randomly chosen initial population values. Parameter values are: $b_S = 0.22$, $b_R = 0.2$, $m = 0$, $\delta_{iS} = 0.8$, $\delta_{iR} = 0.4$ and values for other parameters are given in Table 2. The trivial equilibrium, E_0, is labeled in (a). In (b), the red curve is the sensitive-only mosquito population (M_S) and the blue curve the resistant-only mosquito population (M_R), both curves converge to zero (or to the stable trivial equilibrium) as time increases.



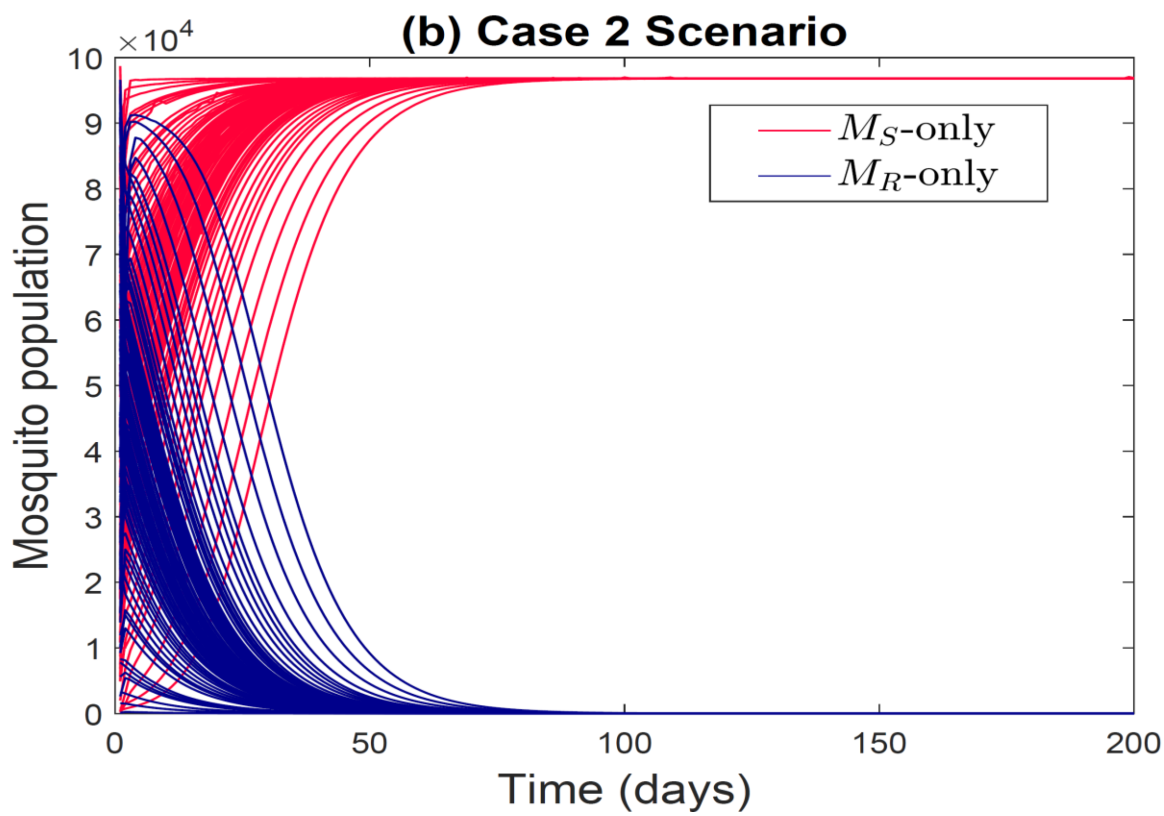


***Figure 3**:* Numerical simulation of the model system (1) for **Case 2 scenario.**

Figure 3 shows numerical simulation of the model system (1) for **Case 2 scenario:** (a) direction field and solution curves and (b) solution curves versus time for 100 randomly chosen initial population values. Parameter values are: $b_S = 4.22$, $b_R = 2.22$, $m = 0$, $\delta_{iS} = 0.07$, $\delta_{iR} = 0.11$ and values for other parameters are given in Table 2. The trivial (E_0), sensitive-only (E_S), and resistant only equilibria (E_R), is labeled in (a). In (b), the red curve is the sensitive-only mosquito population (M_S) and the blue curve the resistant-only mosquito population (M_R), the solution curve converges the stable sensitive-only equilibrium as time increases.

CASE 3 SCENARIO

The values of the thresholds $\mathfrak{R}_M^S$, $\mathfrak{R}_M^R$, and $\mathfrak{R}_M^{Sm}$ for the parameter values in Figure 4 are 24.76, 29.79 and 24.76, respectively. Here too, $\mathfrak{R}_M^S > 1$, $\mathfrak{R}_M^R > 1$ and $m = 0$, and by Theorem 4.2, the trivial (E_0), sensitive-only (E_S) and resistant-only (E_R) equilibria exist, however, the co-existence equilibrium (E_C) does not exist. It is worth to mention that $\mathfrak{R}_M^R > \mathfrak{R}_M^S$ in this scenario, and by Theorem 5.1, the resistant-only equilibrium E_R is stable and the sensitive only equilibrium E_S is unstable. Also, the trivial equilibrium E_0 is unstable (since $\mathfrak{R}_M^S > 1$ and $\mathfrak{R}_M^R > 1$). This result is also confirmed numerically in Figures 4 (a) and (b). The simulation results in Figures 4 (a) & (b) show that the resistant-only equilibrium (E_R) is stable, that is, a simulation with randomly chosen initial value in the interior of the first quadrant of the M_S-M_R plane (and in the positive M_R-axis) converge to the resistant-only equilibrium. If the initial population is in the M_S-axis, the simulation converges to the sensitive-only equilibrium (Figure 4 (a) & (b)). That is, the M_S-axis is a stable manifold of the sensitive-only equilibrium (E_S) (which is unstable). The biological implication of these results is that if small resistant populations are introduced, the chemical insecticide-based mosquito control strategy eventually causes a 100% resistant mosquito population in the environment. Furthermore, the control strategy in this scenario causes a phenomenon that resistant mosquito drives sensitive mosquito to extinction (observe that $\mathfrak{R}_M^R > \mathfrak{R}_M^S$).

CASE 4 SCENARIO

In this scenario, the values of the thresholds $\mathfrak{R}_M^S$, $\mathfrak{R}_M^R$, and $\mathfrak{R}_M^{Sm}$ are 24.76, 18, and 24.62, respectively. By Theorem 4.2, since $m > 0$, $\mathfrak{R}_M^{Sm} > 1$ and $\mathfrak{R}_M^{Sm} > \mathfrak{R}_M^R$, the trivial ($E_0$), resistant-only ($E_R$) and the co-existence ($E_C$) equilibria exist, however, the sensitive-only (E_S) does not exist (since $m > 0$). Figure 5 (a) show that all solution curves with initital values in the interior of the first quadrant in the M_S-M_R plane (and in the positive M_S-axis) converges to the co-existence equilibrium (E_C). Furthermore, if the initial value is in the positive M_R-axis, the solution curve converges to the resistant-only equilibrium (E_R) (Figure 5 (a)). In Figure 5 (b), we run simulations for 100 randomly chosen initial values, the simulations were run for 400 days. Here too, for all chosen initial values, the solution converges to the co-existence equilibrium (E_C) (Figure 5 (b)). The biological implication of these results is that if the initial population consists sensitive and resistance mosquitoes, the chemical insecticide-based mosquito control strategy in this scenario causes the co-existence of the sensitive and resistant mosquito population in the environment for randomly chosen initial population sizes of the sensitive-type and resistant-type mosquitoes.

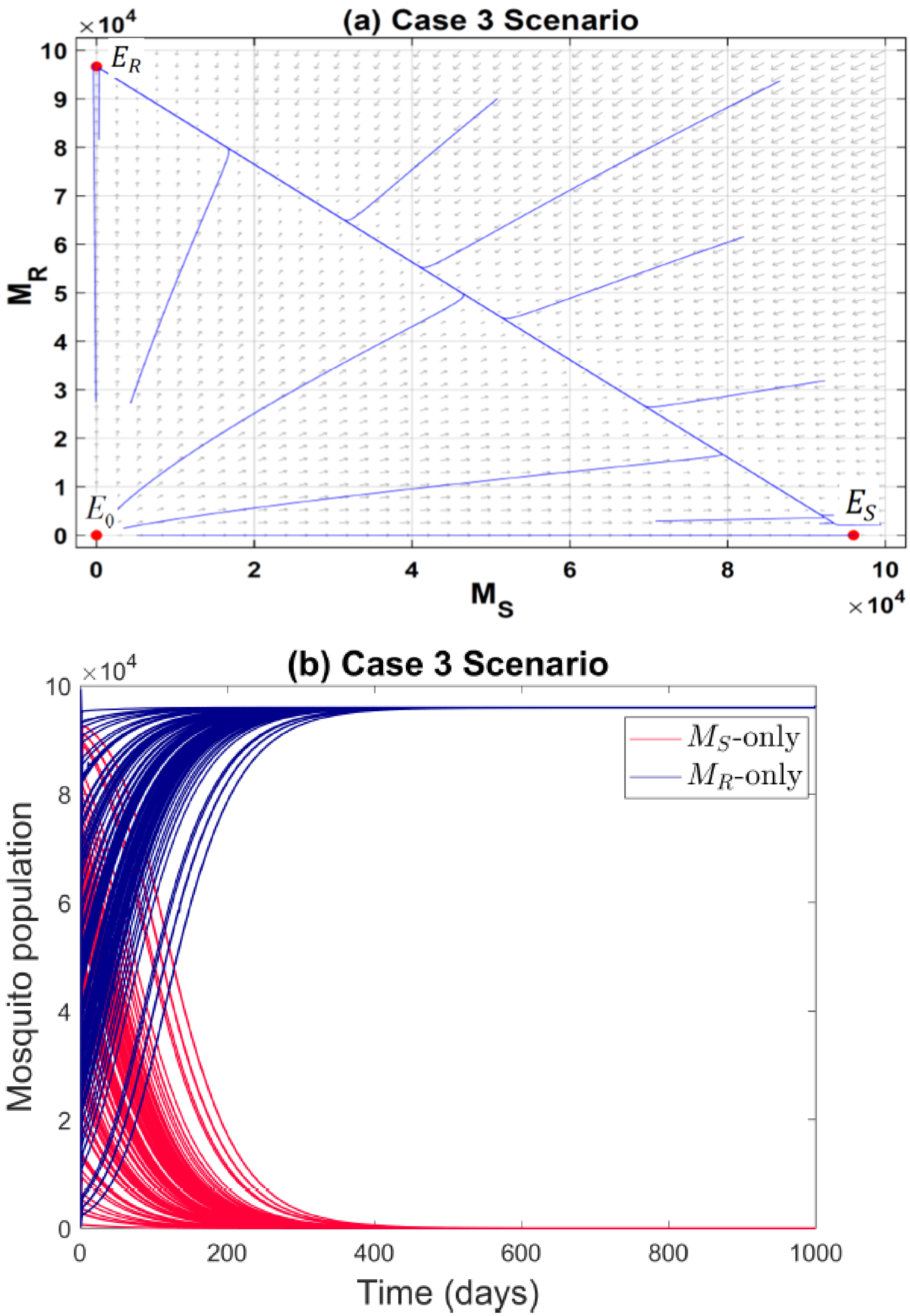


***Figure 4**:* Direction field and solution curves of the model (1) for **Case 3 scenario**.

Figure 4 shows direction field and solution curves of the model (1) for **Case 3 scenario**. Parameter values are: $b_S = 2.22$, $b_R = 4.22$, $m = 0$, $\delta_{iS} = 0.04$, $\delta_{iR} = 0.11$ and values for other parameters are given in Table 2. The trivial (E_0), sensitive-only (E_S), and resistant only equilibria (E_R), is labeled in (a). In (b), the red curve is the sensitive-only mosquito population (M_S) and the blue curve the resistant-only mosquito population (M_R), the solution curve converges to the stable resistant-only equilibrium.

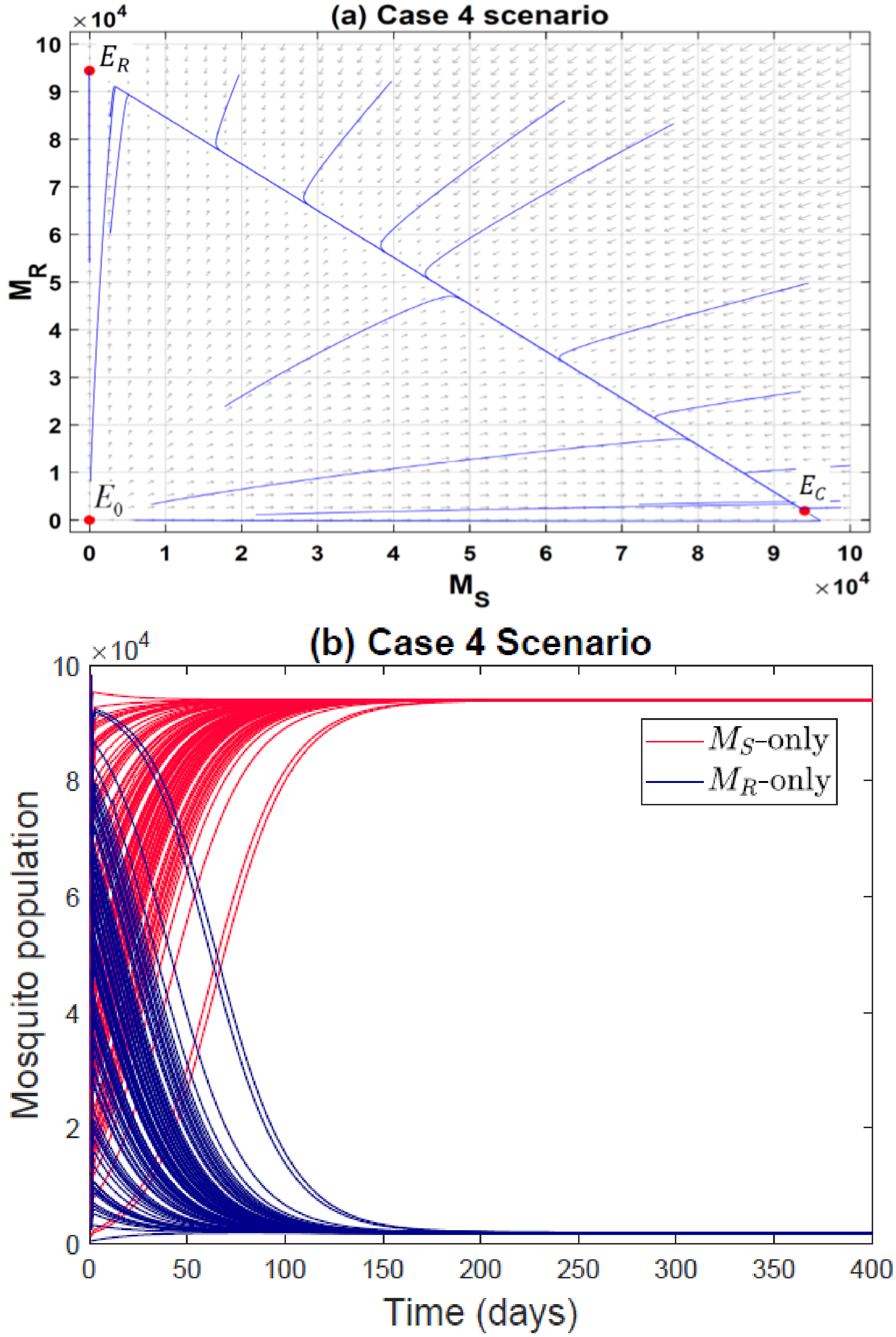


***Figure 5**:* Direction field and solution curves of the model (1) for **Case 4 scenario.**

Figure 5 shows direction field and solution curves of the model (1) for **Case 4 scenario**. Parameter values are: $b_S = 4.22$, $b_R = 3.22$, $m = 0.002$, $\delta_{iS} = 0.11$, $\delta_{iR} = 0.04$, $\mu_R = 1/7$ and values for other parameters are given in Table 2. The trivial (E_0), resistant-only (E_R), and co-existence equilibria (E_C), is labeled in (a). In (b), the red curve is the sensitive-only mosquito population (M_S) and the blue curve the resistant-only mosquito population (M_R), the solution curve converges to the stable co-existence equilibrium as time increases.

CONCLUSION

In this study, we developed a population genetics mathematical model for mosquitoes where insecticide-based mosquito control and mutation from insecticide sensitive to insecticide resistant mosquito are incorporated. We calculated the sensitive-only, resistance-only and co-existence equilibria in terms of the model parameters. Furthermore, we derived the conditions for the existence of four equilibria, that are, trivia (all mosquitos die), sensitive-only, resistance-only and co-existence equilibria. Moreover, we established conditions for the local stability of the trivial (E_0), the sensitive only (E_S) and the resistant only (E_R) equilibria.

We explored four possible scenarios numerically: (1) a scenario in which the chemical insecticide eventually eliminates mosquitoes from the environment, that is, the trivial equilibrium is stable, (2) a scenario in which only sensitive mosquitoes survive in the environment and the resistant ones die out, that is, the sensitive-only equilibrium is stable and the other equilibria are unstable, (3) a scenario in which the use of chemical insecticide eventually cause only resistant mosquitoes to survive in the, that is resistant-only equilibrium is stable and the other equilibria are unstable, and (4) a scenario in which both sensitive and resistant mosquitoes survive in the environment for numerous arbitrary chosen initial conditions.

In conclusion, numerical results show that in scenarios 1 & 2 insecticide resistance is managed and in scenarios 3 & 4 insecticide resistance is not managed. However, in all scenarios, our numerical results show that the totals mosquito population abundance in the environment (at equilibrium) is at the carrying capacity level. Our results suggest the need for further modeling study that incorporates field/experimental data, for example, to realistically estimate the model parameters so that the model can be used to get insights from the model predictions.

ACKNOWLEDGMENTS

Jemal Mohammed-Awel acknowledges the support, in part, of the National Science Foundation (NSF) (DMS-2052355).

References

Centers for Disease Control and Prevention. 2020. Mosquitoes in the United States. https://www.cdc.gov/mosquitoes/about/mosquitoes-in-the-us.html.

Environmental Protection Agency. 2021. Mosquito life cycle. https://www.epa.gov/mosquitocontrol/mosquito-life-cycle.

Gates B. 2014. The deadliest animal in the world, Mosquito Week. https://www.gatesnotes.com/health/most-lethal-animal-mosquito-week.

Lee H., S. Halverson, and N. Ezinwa. 2018. Mosquito-borne diseases. Primary Care, 45(3), 393-407. doi:10.1016/j.pop.2018.05.001 O

Macdonald G. 1957. The epidemiology and control of malaria. Oxford University Press.

Mohammed-Awel J. and E. Numfor. 2017. Optimal insecticide treated bed-net coverage and malaria treatment in a malaria-HIV co-infection model. Journal of Biological Dynamics, 11, 160-191.

Mohammed-Awel J., F. Agusto, R. Mickens, and A. Gumel. 2018. Mathematical assessment of the role of vector insecticide resistance and feeding/resting behavior on malaria transmission dynamics: optimal control analysis. Infectious Disease Modelling, 3, 301-321.

Mohammed-Awel J. and A.B. Gumel. 2019. Mathematics of an epidemiology-genetics model for assessing the role of insecticides resistance on malaria transmission dynamics. Mathematical Biosciences, 3, 33-49.

Morgan N. and G. Robert. 2015. Linearization and stability analysis of nonlinear problems. Rose-Hulman Undergraduate Mathematics Journal, 16(2), Article 5.

Price J. N. 2019. Mosquito-borne illnesses and their disproportionate impact on variant infrastructures: a GIS map comparison of three regions. Eastern Kentucky University Honors Theses. https://encompass.eku.edu/cgi/viewcontent.cgi?article=1690&context=honors_theses

Ross D. and G. Robert. 1916. An application of the theory of probabilities to the study of a priori pathometry. Proceedings of The Royal Society, Series A. 92, 204-230.

World Health Organization. 2022. Malaria world report. https://www.who.int/publications/i/item/9789240064898.